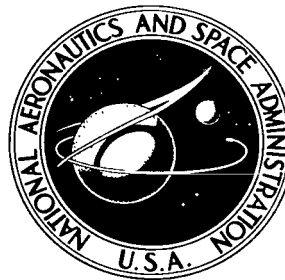


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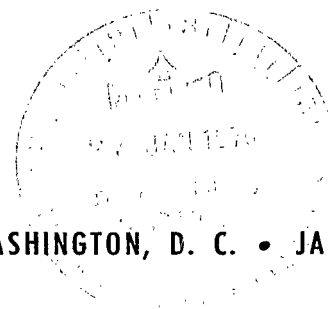


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THERMAL CONDUCTANCE OF AND HEAT GENERATION IN TIRE-PAVEMENT INTERFACE AND EFFECT ON AIRCRAFT BRAKING

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16. Abstract A finite-difference analysis was performed on temperature records obtained from a free rolling automotive tire and from pavement surface. A high thermal contact conductance between tire and asphalt was found on a statistical basis, probably about $3 \times 10^4 \text{ W/(m}^2\text{)(K)}$ or greater. Average slip due to squirming between tire and asphalt was about 1.5 mm. Consequent friction heat was estimated as 64 percent of total power absorbed by bias-ply, belted tire. Extrapolation of results to aircraft tire indicates potential braking improvement by even moderate increase of heat absorbing capacity of runway surface.					
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THERMAL CONDUCTANCE OF AND HEAT GENERATION IN TIRE-PAVEMENT INTERFACE AND EFFECT ON AIRCRAFT BRAKING

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SUMMARY

A finite-difference analysis was conducted on data provided by the University of Michigan, which was obtained during research conducted under an NASA grant. The data consisted of temperatures recorded by (1) thermocouples embedded within the tread of a free-rolling automotive tire and (2) nickel-grid resistance thermometers cemented to the pavement surface and traversed by the tire. Tires used were smooth-tread, bias-ply, belted.

The analysis yielded values of heat generated by friction between the free-rolling tire and the asphalt surface, heat generated by mechanical hysteresis of the rubber, and a lower limit value of thermal contact conductance between tire and asphalt surface. An average element of tread surface area on the tire for an automobile of intermediate weight appeared to slide a total distance of 1.5 mm during a single contact with the asphalt. The friction heat generated by this sliding accounted for about 64 percent of the total road resistance after thermal equilibrium was reached. For the cold tire, the road resistance was predominantly caused by hysteresis of the rubber. Thermal contact conductance between the tire and asphalt was found to be about $3 \times 10^4 \text{ W}/(\text{m}^2)(\text{K})$ or greater. Extrapolation of the results to apply to an aircraft tire suggests that the local slippages between a nonskidding aircraft tire and the runway could well generate enough heat to soften the rubber and thus lower the level of braking force necessary to cause skid. It is suggested that a moderate improvement of heat absorbing capacity of the runway surface might substantially increase the maximum attainable braking effort.

INTRODUCTION

In theoretical study, conducted during the period 1970 to 1972, and concerned with possible use of an aluminum skin on a runway surface for improvement of aircraft brak-

ing (ref. 1), a need was encountered for a value of thermal contact conductance between the tread of a rolling tire and the road, or runway, surface. The authors of reference 1 were unable to find such a value in the literature. Because of the unavailability of such a value, the analysis reported in reference 1 had to be somewhat less complete than desired.

During the same time period, under NASA grant NGR 23-005-417, research was performed at the University of Michigan directed in part toward obtaining at least an approximate value, or a limit value, of the thermal contact conductance. Parts of the results of that research, monitored by the author of this report, were published in reference 2. Some of the data reported in reference 2 served as the basis for the analysis reported here. The data used were of two types: (1) temperature records obtained from thin-film sensors of very short response time, which were cemented to the pavement surface, and over which the test tire rolled at 22.4 m/sec (50 mph) and (2) temperature records obtained from thermocouples embedded at different depths within the tread of the rolling tire.

A finite-difference analysis of the heat flows necessary to create the temperature histories indicated by the records obtained from the thin-film sensors was expected to yield histories of (1) surface temperature of sensor, (2) surface temperature of rubber, (3) rate of heat flow from rubber to sensor or the reverse, (4) heat flow into sensor produced by friction within rubber-sensor interface, and (5) heat flow into rubber produced by friction within rubber-sensor interface. From the thermocouple readings within the tire tread, the rate of heat generated by hysteresis within the tread could be readily calculated. The sum of the heats generated by friction and by hysteresis could be checked for agreement with the power necessary to overcome road resistance of the tire.

From the results described, an approximate lower limit value could be deduced for thermal contact conductance of the tire-sensor interface, which could be converted to a value for the tire-asphalt interface. The rate of heat generation by sliding friction within the interface could also be deduced.

With the thermal contact conductance even approximately known, the thermal aspects of the braking concepts described in reference 1 can be more accurately predicted. Values of both the thermal contact conductance and the rate of heat generation by sliding friction are basic information of interest to the tire industry.

U. S. Customary units were used for principal measurements and calculations and then converted to SI units for reporting purposes.

After this report was written, but not yet published, and after the author's retirement, an additional publication (ref. 3) came to his attention, which was also concerned with research at The University of Michigan under NASA Grant NGR 23-005-417. Analysis and discussion of research data in reference 3 are pertinent to the subject matter of this report. However, rather than to make extensive insertions into the body of this

report in consequence of reference 3, the author decided to treat that reference in an appendix of this report.

SYMBOLS

A_{rd}	total road surface area covered by tire tread per second, m^2 (Personal communication from R. J. Staples of the University of Michigan)
A_{tr}	median of inner and outer surface areas of tread, 2858 cm^2 (calculated)
A_1	inner surface area of tire tread, 2760 cm^2 (less than outer surface area because of smaller diameter) (Personal communication from R. J. Staples of the University of Michigan)
A_2	outer surface area of tire tread, 2960 cm^2 (Personal communication from R. J. Staples of the University of Michigan)
c	specific heat, $J/(g)(K)$
c_{as}	specific heat of asphalt, 1.727 J/(g)(K) calculated from value of ρ_{as} with equation from ref. 4)
c_f	coefficient of friction
c_{ni}	specific heat of nickel, 0.431 J/(g)(K) (ref. 5)
c_{pol}	specific heat of polyimide, 1.094 J/(g)(K) (value supplied by manufacturer)
d	tread thickness, 23 mm (estimated from data in ref. 2)
$g(t)$	rate of heat generation by friction per unit area within interface of rubber and supporting surface relative to which it slides at time t , W/m^2
H_{hi}	average rate of hysteretic heat generation within tread during time interval i , W
H_{oi}	average rate of flow of hysteretic heat out of tread during time interval i , W
H_{oj}	rate of flow of hysteretic heat out of tread at time j , W
h	thermal contact conductance between surfaces in contact, $W/(m^2)(K)$
h_{as}	reciprocal of contact resistance of asphalt surface as defined by eq. (22), $W/(m^2)(K)$
$h_{j,j+1}$	thermal contact conductance of interface between layers j and $j + 1$ for use in finite difference equation, $W/(m^2)(K)$
h_{p-ni}	thermal contact conductance between polyimide and nickel grid in resistance thermometers, $4800 \text{ W/(m}^2)(K)$ (calculated)

h_{pol}	reciprocal of contact resistance of polyimide surface as defined by eqs. (20) and (21), $W/(m^2)(K)$
h_{rs}	reciprocal of contact resistance of rubber surface as defined by eqs. (20) and (21), $W/(m^2)(K)$
$K_{j,j+1}$	conductance factor defined by eq. (A5), W/K
k	thermal conductivity of an unspecified substance, $W/(m)(K)$
k_{as}	thermal conductivity of asphalt, $0.111 W/(m)(K)$ (calculated from value of ρ_{as} with equation from ref. 4)
k_j	thermal conductivity of material in layer j for use in finite difference equation, $W/(m)(K)$
k_{ni}	thermal conductivity of nickel, $59.5 W/(m)(K)$ (ref. 5)
k_{pol}	thermal conductivity of polyimide, $0.156 W/(m)(K)$ (value supplied by manufacturer)
k_r	thermal conductivity of tread rubber, $0.27 W/(m)(K)$ (determined by a commercial testing laboratory)
k_{rs}	thermal conductivity of rubber surface, $0.14 W/(m)(K)$ (value for acrylonitrile butadiene styrene from ref. 6)
m_{1j}	numerical value of slope of temperature curve at inner surface of tread at time j (fig. 14)
m_{2j}	numerical value of slope of temperature curve at outer surface of tread at time j (fig. 14)
p	normal pressure between tread and supporting surface, N/m^2
R_f	road resistance to overcome friction between rubber and supporting surface, N
s	average distance of slip, m
$\overline{\Delta T_i}$	average temperature rise within tread during time interval i , K
t	time, sec
Δt	time interval between one time point and another in finite-difference equation, sec
Δt_i	duration of time interval i , sec
t_0	time of start of contact of tire with sensor
$u(0, t)$	surface temperature of unspecified material at time t , K
$u_{j,n}$	temperature of midpoint of layer j of unspecified material at time point n in finite-difference equation, K

$u_{0, n}$	surface temperature at time point n from finite difference solution
$u_{0, p(n, n+1)}$	average temperature of polyimide surface during interval from time point n to time point $n + 1$
$u_{0, r(n, n+1)}$	average temperature of rubber surface during interval from time point n to time point $n + 1$
$u_{0(n, n+1)}$	average temperature of surface of unspecified material during interval from time point n to time point $n + 1$
$u_{pol}(x, t)$	temperature at distance x below surface of polyimide at time t , $0 \leq x < \infty$, K
$u_r(x, t)$	temperature at distance x below surface of rubber at time t , $0 \leq x < \infty$, K
W_f	friction work, J
Δx	thickness of layer of material for use in finite-difference equation when all layers are of same thickness, mm
$(\Delta x)_j$	thickness of layer j in finite-difference equation, mm
$(\Delta x)_n$	thickness of n^{th} layer for finite-difference program, mm
α	thermal diffusivity of unspecified material, m^2/sec
α_{as}	thermal diffusivity of asphalt, $6.15 \times 10^{-8} m^2/sec$ (calculated)
α_{pol}	thermal diffusivity of polyimide, $1.004 \times 10^{-7} m^2/sec$ (calculated)
α_r	thermal diffusivity of tread rubber, $1.11 \times 10^{-7} m^2/sec$ (determined by commercial testing laboratory)
ρ	density of an unspecified substance, g/m^3
ρ_{as}	density of asphalt, $1.045 \times 10^6 g/m^3$ (average value from ref. 4)
ρ_{ni}	density of nickel, $8.9 \times 10^6 g/m^3$ (ref. 5)
ρ_{pol}	density of polyimide, $1.42 \times 10^6 g/m^3$ (value supplied by manufacturer)
φ	constant rate of thermal flux through surface into unspecified material, W/m^2
$\varphi_{c(n, n+1)}$	conductive thermal flux through contact conductance from polyimide surface to rubber surface during interval from time point n to time point $n + 1$, W/m^2
$\varphi_{f(n, n+1)}$	rate of generation of heat by friction during interval from time point n to time point $n + 1$, W/m^2

$\varphi_{p(n, n+1)}$	average thermal flux through uppermost surface of polyimide in finite-difference equation during interval from time point n to time point $n + 1$, W/m^2
$\varphi_{r(n, n+1)}$	average thermal flux through outermost surface of rubber in finite-difference equation during interval from time point n to time point $n + 1$, W/m^2
$\varphi_{rf(n, n+1)}$	rate of generation of heat by friction on rubber surface, according to equivalent imaginary situation described in discussion of eq. (1), during interval from time point n to time point $n + 1$, W/m^2

THERMAL ANALYSIS

It was possible to deduce much about the frictional heat generation within and flow of heat through the interface of smooth-tread tires and pavement by analysis of records from thin-film temperature sensors cemented to the pavement surface. Such analysis made use of a finite-difference computer program. Within this program, the tread rubber, the thin-film sensor, and the upper part of the underlying asphalt were subdivided into imaginary layers. The heat flow from layer to layer and the heat generated on the surface were then calculated.

Temperature records from thermocouples embedded within the tire tread were also analyzed in a fairly conventional manner to determine heat generated by hysteresis.

Comparison of results of the two analyses, hopefully, might indicate that each result is of a reasonable magnitude, or that one or the other is not reasonable. The analysis of records from the thin-film sensors will be discussed first.

THERMAL BEHAVIOR OF TIRE-PAVEMENT INTERFACE

Figure 1 is a reproduction of figure 7 of reference 2. It is a schematic sketch of a temperature sensor (a resistance thermometer), four of which were cemented to the pavement surface side by side. Only two were used for the records that will be analyzed here. The centers of those two sensors were 63 mm apart on the pavement, in the direction at right angles to that of tire movement. A nickel grid in each sensor, 0.005 mm thick, formed one continuous conductor. Changes in its temperature changed its electrical resistance, which was continuously measured with a Wheatstone bridge circuit, and recorded with a cathode-ray oscilloscope. The electrical system provided approximately 98 percent response to a step function within 0.2 msec (private communication from R. J. Staples of The University of Michigan). The nickel grid was em-

bedded between an upper polyimide sheet of 0.0127 mm thickness and a lower polyimide sheet of 0.0254 mm thickness. The manufacturer of the sensors supplied values as shown in the section SYMBOLS for the thermal properties of the polyimide, conductivity k_{pol} , density ρ_{pol} , and specific heat c_{pol} . From these values, the value of thermal diffusivity α_{pol} as shown in the section SYMBOLS was computed.

Temperature-time records from the nickel-grid sensors used in this analysis are shown in figures 2 and 3, which were taken from reference 2. Test data for those records were as shown in the tabulation in the following table:

Trace	Figure	Temperature sensor	Distance of travel of tire in contact with pavement before touching sensor, km	Distance of sensor from centerline of tread during contact, mm	Measured temperatures, K		
					Air	Road surface	Tire surface
1	2	2	0.03 or less	24	292.6	299.1	296.5
2	2	3	.03 or less	39	292.6	297.1	296.5
3	3	2	1.6 to 3.2	5	293.7	298.2	302.1
4	3	3	1.6 to 3.2	58	293.7	298.2	302.1

Traces 1 and 2 were made simultaneously by one and the same tire, as were traces 3 and 4. The tires used for the records of both figures 2 and 3 were standard automotive tires, cross biased, belted, and designed to carry approximately 4450 N (1000 lb) load. They traversed the sensors while bearing that load at 22.35 m/sec (50 mph). Inflation pressure was 1.65×10^5 N/m² (24 psi). The temperatures shown in the tabulation were taken at the times of the tests or soon afterward. The road surface and tire surface temperatures were measured with contact pyrometers.

The four records in figures 2 and 3 were analyzed by simple finite difference methods in two stages. The first stage concerned itself only with the thermal fluxes and temperatures (1) on the upper surfaces of the sensors, (2) within the polyimide and nickel of the sensors, and (3) within the underlying asphalt. The second stage was concerned with thermal flux and temperatures (1) on the surface of the tire, (2) within the tire tread, and (3) on the upper surfaces only of the sensors. The first of these two stages of the analysis will now be described.

Thermal Fluxes and Temperatures in Nickel-Grid Sensors and Asphalt

Figure 4 is a greatly enlarged sectional view showing part of the interface between tire tread and temperature sensor with an imaginary barrier between them to represent thermal contact resistance. Also shown are interfaces between some of the layers into which the materials were subdivided for the purpose of numerical analysis. All interfaces are seen in the figure on edge. Various entities that will be used in description of the analysis are identified in this figure.

All parts of each nickel-grid sensor are assumed to have been at the same temperature at time t_0 when the tire surface first came into contact with it. The problem would be simpler if a history of thermal flux through the upper polyimide surface after time t_0 were known. If so, the resulting temperature-time records for all levels, including both the upper polyimide surface and the nickel grid, could be calculated by the finite-difference technique described in appendix A. But the problem here is the inverse. The temperature-time record of the nickel grid is given. The flux history through the uppermost polyimide surface is unknown and needs to be found.

A flux history through the polyimide surface can be assumed. The resulting temperature-time record of the nickel grid can be calculated. This result can be compared with the given record, and the flux history can be revised in a manner designed to reduce the difference. This procedure can be iterated, starting each iteration, not with an assumed flux history, but with the revised flux history from the preceding iteration. This iterative procedure involved the principal difficulty encountered in the analysis. An automatic way was needed to modify the previously assumed or revised flux history that would always cause the resulting nickel-grid temperature history to approach more closely to the actual record throughout its length. The successful method that was developed and used will not be described here because it is of only academic interest relative to the purpose of this report. The flux histories that were eventually derived stand on their own without regard to their method of derivation, because they do produce the correct nickel-grid temperature histories by the finite-difference method.

In the application of the finite-difference method to the problem of finding flux histories, the values of conductivity, density, and specific heat given in the section SYMBOLS for nickel and polyimide were used (k , ρ , and c with subscripts ni and pol). Substantially different values of the thermal constants for asphalt from those given in the section SYMBOLS (k_{as} , ρ_{as} , and c_{as}) were used. However, the results from operation of the program indicated that only negligible heat got through both layers of polyimide and the nickel grid to the asphalt. So, when the values of the thermal constants for asphalt were later corrected, it was thought to be useless to rerun the program.

In the operation of the finite-difference computer program, the upper layer of polyimide, 0.0127 mm thick (see fig. 1), was divided into five layers of uniform thickness, as was the lower layer, 0.0254 mm thick. The nickel grid was treated as a single

layer, 0.005 mm thick, and was treated as a sheet rather than a grid. The upper part of the asphalt underlying the sensor was divided into layers with uniform thickness of 0.0127 mm. As many layers were used as needed, for any particular run of the program, so that the maximum variation of temperature of the lowest layer was not greater than about 5×10^{-5} K relative to the initial temperature assumed the same for all layers. In early runs of the computer program, thermal resistance between either layer of polyimide and the nickel grid was assumed to be zero (conductance infinite). In final runs, however, for a reason that will be explained during description of the results of the program, contact thermal conductance between polyimide and nickel grid, on each side of the nickel grid, was taken as the value of h_{p-ni} shown in the section SYMBOLS. The cement layer holding the sensor to the pavement was treated as infinitely thin and as providing infinite conductance between polyimide and asphalt. Time increments for which calculations were made were of uniform duration of 31.25 μ sec. Readings of temperature levels from the four records shown in figures 2 and 3 were made, with use of greatly enlarged copies of those figures, only for intervals of 125 μ sec. Hence, three out of four of the temperature values used for the 31.25- μ sec intervals were interpolated.

The results from the completed first stage of the analysis of the temperature records seen in figures 2 and 3 are shown in figures 5 to 8. Curves relative to time are shown in each of those figures for (1) temperature of the uppermost polyimide surface of sensor, (2) thermal flux into sensor through uppermost polyimide surface, and (3) the temperature of the nickel grid in the sensor. The curves for temperature of the nickel grid are just enlarged tracings from the records shown in figures 2 and 3. The plotted circular points are nickel grid temperatures calculated by the finite-difference program from the curves of thermal flux through the uppermost surface of the sensor. A point is plotted for each time at which the numerical value of the difference between the calculated temperature and the temperature previously read from the curve passed through a maximum. In a few cases, additional points were plotted.

Note that the curves for thermal flux for traces 2, 3, and 4 (figs. 6 to 8) eventually level off at a value substantially zero. They do so early enough to permit an assumption that the tire left contact with the sensor at the point in time where such leveling off occurred. For trace 1 (fig. 5) fluctuations of thermal flux continued long after the tire must have left contact, but eventually the flux did level off substantially at zero. The flux traces in figures 5 and 7 both show some oscillations toward the ends of the traces after having remained substantially level at zero for some distance. These oscillations at the ends of the traces are thought to be due to an imperfection in the method of adjustment of the flux traces after each iteration of the computer program. It operated well except near the end of each trace, where thousands of iterations would apparently have been necessary for good convergence.

Eventual leveling of all flux curves at zero was made possible by the previously mentioned assumption of a contact conductance of $4800 \text{ W}/(\text{m}^2)(\text{K})$ for each interface between nickel grid and polyimide. Thermal flux curves analogous to those of figures 5 to 8, not reproduced here, were obtained by computer runs with use of higher and lower assumed values of this contact conductance. Such curves did not level off completely but appeared to approach zero asymptotically. From tire dimensions and vehicle speed the tire must have left contact with the sensor at some time well within the duration of each record. Hence, the behavior of the flux curves with values of the contact conductance greater or less than $4800 \text{ W}/(\text{m}^2)(\text{K})$ could not be realistic. The value of $4800 \text{ W}/(\text{m}^2)(\text{K})$ was therefore assumed to be approximately correct.

For trace 1 in figure 5, however, the fluctuations of thermal flux throughout the time interval from about 8.5 to about 16 msec must be presumed to be real, despite the fact the tire must have left contact at about the beginning of this interval. Many possible explanations exist. The oscillations could possibly be explained by simple errors in reading of ordinates from the temperature trace in figure 2. The fact remains, however, that these oscillations do not exist for the other three traces. The ordinate readings were made by the same person in the same manner. Perhaps the most probable explanation is that some contamination existed between the tire surface and the surface of the sensor. If such contamination were, for instance, a minute solid particle that slid or rolled across the surface of the sensor for about 7.5 msec after the tire left contact, the oscillations of flux through the upper surface of polyimide would be explained. Also, such a particle could displace the tread rubber in such a way as to cause a shorter than normal contact of rubber with the sensor surface, which may actually have happened, as will be discussed later.

The authors of reference 2 found that the contact patch of the stationary tire under load was about 14 cm (5.5 in.) long and concluded, therefore, that the duration of contact must be 6.25 msec. But in all the plots of thermal flux through upper surface into sensor in figures 5 to 8 the interval throughout which substantial fluctuations of flux existed was obviously greater than 6.25 msec. It appears that hysteretic resistance to changes in tread shape must have caused the contact patch to be appreciably longer than 14 cm (5.5 in.) for the rolling tire. An estimate of the actual duration of contact for each of the four traces will be required later. Making of such estimates, however, will be deferred until needed.

The appearances of all the thermal flux curves indicate a stick-slip condition between rubber and polyimide. When slip occurred, heat was generated sharply by friction, and high peaks occurred in the flux curves. When sticking occurred, no friction heat was generated, and conduction of heat from polyimide to rubber caused sharp declines or even negative fluxes into the polyimide through the uppermost surface.

The curves representing temperature of uppermost sensor surface and thermal flux through uppermost surface of sensor, in conjunction with assumptions of (1) contact

thermal conductance between polyimide and rubber and (2) initial temperature of rubber relative to polyimide surface, are sufficient for calculation of the thermal history of the rubber including temperatures and thermal fluxes at all depths, as well as rates of generation of heat by friction. At this stage no basis exists for either of the two assumptions required. However, the calculations can be made for each of a series of combinations of the two assumptions, and the results may be analyzed for plausibility under the various combinations of assumptions. Such a procedure and its results will now be discussed.

Thermal Fluxes and Temperatures of Tire Tread and Rates of Generation of Heat by Friction

The second stage of analysis of the four records in figures 2 and 3, concerning heat flow into the tire tread, was a more complex problem than the flow into the nickel-grid sensors. The computational basis will now be described; then the computational results will be presented.

Computational basis and theory for heat flow into tire tread. - Sharp temperature gradients may be expected at very small depths into a surface element of the tire tread during its contact with the pavement. It was assumed that these gradients will almost completely disappear during the much longer time interval throughout which the surface element is in contact only with air before the next contact with the pavement. During this much longer period, the material very close to the surface loses heat both to the air and to the rubber deeper under the surface. Loss of rubber from tread surface was assumed negligible.

The finite difference method applied to the tire tread was a modified form of that which was applied to the sensor in the first stage of the analysis. Details are described in appendix A. In keeping with the content of the preceding paragraph, uniform initial temperature of rubber was assumed as in the application of the method to the temperature sensor. The necessary modifications were due to the need for including the effect of thermal contact conductance between polyimide and rubber and the need for separating the effects of conductive heat flux through the polyimide-rubber interface from the effects of heat generated by friction within the interface.

A theoretical equation, analytically derived in reference 1, was used for the separation of conductive heat flux from the heat generated by friction. A one-dimensional form of that equation is

$$-k_{rs} \frac{\partial u_r(x, t)}{\partial x} = \left(\frac{k_{rs}}{k_{rs} + k_{pol}} \right) g(t) + h[u_{pol}(x, t) - u_r(x, t)] \quad x = 0 \quad (1)$$

where k_{rs} is thermal conductivity of the rubber surface, $u_r(x, t)$ and $u_{pol}(x, t)$ are temperatures within or on the surface of rubber and polyimide, $g(t)$ is the rate of heat generation by friction, and h is thermal contact conductance. (Derivation of this equation in ref. 1 required the assumption that asperities on the rubbing surfaces remained in contact long enough that steady-state conditions would exist within each asperity throughout most of the contact time. Unfortunately, that assumption was not stated in ref. 1.) This equation says that the nature of heat generation and flow is equivalent to an imaginary situation in which two processes occur simultaneously: (1) heat generation by friction occurs independently on each of the two rubbing surfaces in proportion to the conductivities of the materials and (2) heat flows from one surface to the other at a rate determined by their temperature difference and the contact conductance in the same way as if the two materials were not sliding relative to each other.

It is shown in reference 1 that equation (1) applies without regard to the presence of microscopic areas within which the materials are separated by contaminants, provided only negligible heat is generated by friction in the sliding of either material on contaminant or in shearing action within the contaminant itself. In application of equation (1) here, it was assumed that sliding of carbon black over polyimide would involve negligible friction as compared with the sliding of rubber over polyimide. It was also assumed that grains of carbon black on the tread surface would tend to be lost quickly from the rubber matrix. For those reasons, the surface conductivity of tread k_{rs} was taken as 0.14 W/(m)(K) or 0.08 Btu/(ft)(hr)(°F) rather than a considerably higher value k_r that would apply to the tread rubber containing a large quantity of carbon black. This value is the minimum given for a rubber-like plastic (acrylonitrile butadiene styrene) in reference 6. The lowest supportable value was used because its use reduced to a minimum a theoretical anomaly in the overall results, which will be mentioned in the discussion of those results. Use of the value of k_{rs} in equation (1) rather than k_r is speculative. However, as will be explained in discussion of results, use of the value k_{rs} is thought to be conservative in relation to the most important of the results obtained. For the tread rubber exclusive of its surface, that is, for all purposes except use in equation (1), the conductivity k_r was taken as 0.28 W/(m)(K). Thermal diffusivity of the rubber α_r was taken as 1.18×10^{-7} m²/sec. After this study was completed, actual tests by a commercial testing laboratory on the tread rubber indicated conductivity of 0.27 W/(m)(K) and diffusivity of 1.11×10^{-7} m²/sec. These differences were believed too small to justify repeating the work.

In use of the finite-difference computer program for the tread the thickness of the n^{th} layer from the surface was

$$(\Delta x)_n = \exp \left(\ln 2.54 \times 10^{-4} + \frac{n-1}{79} \ln 100 \right) \quad (2)$$

This equation gives a thickness of 2.54×10^{-4} cm (10^{-4} in.) for the surface layer and a thickness of 0.025 cm (0.01 in.) for the 80th layer, with a logarithmic gradation of thickness from layer to layer. The total thickness for the 80 layers was 0.444 cm (0.175 in.). However, only eight layers with a total thickness of about 2.51×10^{-3} cm (about 9.9×10^{-4} in.) were actually used because the program was only allowed to run deep enough to involve a temperature change greater than about 5×10^{-5} K (about 10^{-4} °F) in the deepest layer.

This second-stage analysis, of course, should not be conducted over more than the time interval throughout which the tire tread is judged to have been in contact with the sensor, though appreciable inaccuracy in estimation of beginning and end of contact did not appear to have a large effect on the end result. The following reasoning and procedure were adopted to resolve the problem of estimation of duration of contact:

(1) For each of the four flux curves (figs. 5 to 8), the beginning of contact was taken as being at the time of the first sharp rise of the flux curve above approximately zero value. The results were 1.25, 1.75, 0.75, and 1.625 msec for traces 1 to 4, respectively.

(2) It was recognized that trace 1 would have to be treated as a special case in estimating the time of the end of contact.

(3) Among the other three traces, it was reasoned that trace 4 should have involved the shortest contact time and trace 2 the next shortest, because of the relative distances of the sensors from the centerline of the tire tread during contact.

(4) It was recognized that some subjectivity might exist in favor of a long contact, with consequent great flow of heat through the tire-sensor interface, because the principal purpose of the analysis was concerned with the concept of improving aircraft braking by use of a runway surface having a high thermal conductivity.

(5) To counter such possible subjectivity, it was decided to take the interval of contact for trace 4 as being as short as reasonably tenable. Hence, as shown in figure 8, the end of contact was taken as the time of passage of thermal flux through zero just after the third highest of the peak values, at 9.35 msec, giving a contact duration of 7.725 msec.

(6) The interval of contact for trace 2 was made as short as it reasonably could be made while still being no less than the interval of contact for trace 4. This criterion placed the end of contact as shown in figure 6 at the first crossing of the zero level of the thermal flux curve later than 9.5 msec, which was at 9.75 msec, giving trace 2 a contact interval of 8.0 msec.

(7) The same procedure was followed for trace 3 in comparison with trace 2. The result was termination of contact at 9.125 msec as shown in figure 7, giving a contact interval of 8.375 msec.

(8) As earlier mentioned, it was recognized that if the anomalies in trace 1 were due to a hard contaminating particle, it could displace the tread rubber in a manner to

cause a shorter than normal contact of rubber with the sensor surface.

(9) It was assumed that the general pattern of sticking and slipping between tire tread and supporting surface would be roughly the same in all cases. The flux curves for traces 1 and 3 are more similar than for 1 and 2 or for 1 and 4. It was observed that the thermal flux curves for traces 1 and 3 both showed a fairly uniform flux for a period of about 2 msec during the middle portion of the contact interval. At the end of this period, peaks and valleys were compared in the two records as indicated by the numbers adjacent to those peaks and valleys in the figures. This comparison led to the placing of the end of contact for trace 1 at 8.75 msec, giving a contact duration of 7.5 msec. It was considered that this result could well be wrong, but, at least, in comparison with the more firmly justified contact intervals for the other traces, this result is cautious in relation to the type of possible subjectivity mentioned earlier.

Computational results for tire tread, including tire-sensor interface. - Of the many runs of the finite-difference computer program as applied to the tire surface, only four crucial examples are shown in detail, in figures 9 to 12. Shown in each figure, throughout the contact duration, are (1) the temperature of the rubber surface relative to the initial temperature of the polyimide surface (referred to hereafter as rubber surface temperature), (2) the temperature of the rubber surface relative to the simultaneous temperature of the polyimide surface (referred to hereafter as relative rubber surface temperature), (3) conductive heat flux from polyimide surface to rubber surface (referred to hereafter as conductive flux), and (4) rates of generation of heat by friction on surfaces according to earlier described model representing equation (1) (referred to hereafter as rubber friction heat, polyimide friction heat, and total friction heat). Because the relative friction heat between the two surfaces is analytically treated as proportional to conductivities, one curve suffices for all, but with three different scales.

Figures 9 and 10 are for trace 1 with assumed contact conductances of 11 345 and 56 744 W/(m²)(K) or 2000 and 10 000 Btu/(ft²)(hr)(°F), respectively. Figures 11 and 12 are for the same respective contact conductances, but for trace 2.

Each figure is also for an assumed initial rubber temperature. However, in each case that assumed initial value was reached as a result of numerous repetitions of the entire computer program. An initial value was sought that would result in satisfaction of the conditions that friction heat could never be negative and that friction heat should go to zero at least once and probably several times during the contact interval because of the stick-slip condition. In figure 9, for example, negative values appear in the plot. However, they are only at the minima of sharp downward excursions. It could not be hoped that those minima would be accurate because the nature of the solution is stepwise rather than continuous. What was actually sought was that the lowest average of any five consecutive values of total friction heat should be zero within an arbitrary small value of $\pm 315 \text{ W/m}^2$ (about 100 Btu/(ft²)(hr)). An average of five consecutive values was used because that number proved to be most satisfactory for meeting the conditions that

the average should go substantially to zero several times without ever going substantially negative. The time interval represented by the five consecutive values may well have approximated the time interval most often covered by a sticking condition. For each complete run of the program for an assumed initial rubber temperature, both that initial temperature and the minimum average of five consecutive values of friction heat were recorded. After the second such run, new assumed values were chosen by interpolation or extrapolation relative to the two previous values until the desired accuracy was achieved.

We may now make several evident observations about the plots in figures 9 to 12. In each case the relative rubber surface temperature quickly rises to a level at which it remains fairly constant throughout the rest of the contact interval. The conductive flux quickly drops to a value at which it stays roughly constant, although with the higher value of contact conductance there is a substantial response of the conductive heat flux to peaks and valleys of the heat generation curve. Finally, for a given trace, the general level of both the conductive heat flux and the friction heat are nearly independent of the assumed value of contact conductance throughout the range of values used. These observations apparently mean, for the values of contact conductance assumed:

(1) The difference between rubber and polyimide surface temperatures, after the initial transient condition, is almost solely that required to transmit heat from polyimide to rubber because of the fact the rubber was initially cooler than the polyimide. The contact apparently does not last long enough for this heat flux to diminish appreciably. Hence, an approximately constant temperature difference is involved in driving this heat through the contact resistance.

(2) Rapid fluctuation of friction heat, distributed between the two surfaces according to equation (1), causes almost identical rapid fluctuations of surface temperatures of rubber and polyimide.

The following equation is of interest regarding the observation that rapidly fluctuating temperatures of rubber and sensor surfaces remain always nearly the same:

$$u(0, t) = \frac{2\varphi}{k} \left(\frac{\alpha t}{\pi} \right)^{1/2} \quad (3)$$

This equation is given by Carslaw and Jaeger (ref. 7) for a semi-infinite solid, initially at uniform zero temperature, subjected to a uniform flux through its surface. The symbol $u(0, t)$ represents the surface temperature at time t from the start of the flux, k is the conductivity of the material, φ is the flux, and α is the diffusivity of the material. For the special case covered by this equation, to get the same temperature on both surfaces at the end of any time interval during contact we may equate right sides of equation (3) for rubber and polyimide and solve for φ_r/φ_{pol} , where φ_r and φ_{pol} are fluxes into rubber and polyimide, respectively. That is,

$$\frac{\varphi_r}{\varphi_{pol}} = \frac{k_r}{k_{pol}} \left(\frac{\alpha_{pol}}{\alpha_r} \right)^{1/2} = 1.646 \quad (4)$$

But φ_r/φ_{pol} according to equation (4) is nearly the same as k_r/k_{pol} , because α_{pol} and α_r are nearly the same. But equation (1) distributed the friction heat according to the ratio k_{rs}/k_{pol} , not the ratio k_r/k_{pol} . The observation that the rapidly fluctuating temperatures of the two surfaces always remained about the same, therefore, implies that all the assumed values of contact conductance were so low that it really mattered little whether k_r or k_{rs} was used for the operation of the finite-difference program.

None of the observations mentioned appear to be of any value in estimating the correct value of contact conductance.

In figure 13 are plotted the most significant of the results obtained from not only the four computer runs of figures 9 to 12, but from 18 other runs also. In figure 13 all points are plotted against the assumed value of contact conductance between polyimide and rubber that was used for the pertinent computer run. Values plotted for each trace are (1) initial rubber temperature, (2) average conductive flux throughout interval of contact, and (3) average friction heat. An anomaly exists for trace 2 at an assumed contact conductance of about $3.8 \times 10^4 \text{ W/(m}^2\text{)(K)}$. All the plotted points for that computer run are out of line with the other plotted values for trace 2. The program was rerun for contact conductances of about 3.35×10^4 and $3.95 \times 10^4 \text{ W/(m}^2\text{)(K)}$. These runs yielded values that did fall in line with the other points. It was believed the anomaly was not too serious and the problem was resolved by a decision to use the two sets of values at 3.35×10^4 and $3.95 \times 10^4 \text{ W/(m}^2\text{)(K)}$ and ignore the anomalous solution.

Statistical Indication of Slip Pattern Within Contact Area Between Tire and Pavement

The ascending order for curves of average friction heat in figure 13 is the same as the ascending order of distances from tread centerline, namely, traces 3, 1, 2, and 4. We will examine this relation for its implication regarding the pattern of slip within the entire contact area between tire and pavement.

We will consider here a median contact conductance in figure 13 of about $3.2 \times 10^4 \text{ W/(m}^2\text{)(K)}$. We will assume that all elements of rubber surface will slide during contact through the same distance s_t in the direction of vehicle motion, regardless of their distance from the tread centerline. (See fig. 1 and discussion of that figure in ref. 1.) We will assume that each element also will slide in the direction at right angles to that of vehicle motion, through a distance δs_t , where δ is the distance from the tread centerline and s_t is a constant. This model seems reasonable. There is no ob-

vious way that the distance from the tread centerline can greatly affect the longitudinal slip s_l . The transverse slip δs_t must be zero at the tread centerline ($\delta = 0$) because of symmetry. It seems reasonable to suppose that the transverse slip would increase with the distance δ , though not necessarily proportionally as assumed in this model.

According to this model, the total slip distance will be

$$s = \sqrt{s_l^2 + (\delta s_t)^2} \quad (5)$$

We will use a simplifying approximation that contact pressure is uniform throughout the contact area. Then the friction heat should be proportional to s of equation (5).

If we use for δ in equation (5) the distances of the sensors from the tread centerline for traces 1 to 4 in millimeters, and we make $s_t = 0.05 s_l$, we find from equation (5) for traces 1, 2, 3, and 4, respective slip distances of $1.56 s_l$, $2.10 s_l$, $1.03 s_l$, and $3.07 s_l$. The correlation coefficient between these values of s and the average friction heat at contact conductance of $3.2 \times 10^4 \text{ W/(m}^2\text{)(K)}$ is approximately 0.986.

The linear correlation coefficient between two variables x and y is defined as

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n\sigma_x\sigma_y} \quad (6)$$

where \bar{x} and \bar{y} are the average values of the x and y variables, n is the number of values of each of the variables within the sample, and σ_x and σ_y are the standard deviations of x and y . Equation (6) is equivalent to

$$r = \vec{V}_x \cdot \vec{V}_y \quad (7)$$

where \vec{V}_x and \vec{V}_y are unit vectors in n -dimensional space defined as

$$\vec{V}_x = \sum_{i=1}^n \frac{(x_i - \bar{x})}{\sqrt{n} \sigma_x} \vec{v}_i \quad (8)$$

and

$$\vec{V}_y = \sum_{i=1}^n \frac{(y_i - \bar{y})}{\sqrt{n} \sigma_y} \vec{v}_i \quad (9)$$

and where \vec{v}_i ($i = 1$ to n) are a set of mutually orthogonal unit vectors spanning the n -dimensional space. Thus the correlation coefficient r is the cosine of the angle between the vectors \vec{V}_x and \vec{V}_y , with a maximum value of 1 when the two vectors are identical.

In reference 8 a distribution function (eq. (C17)) was derived for β , the cosine of the angle between two vectors in n -dimensional space, under the assumption that each vector is as likely to extend in any one direction as any other within the n -dimensional space. For four-dimensional space that equation is

$$F(\beta) = \frac{4}{\pi} \int_{\theta}^{\pi/2} \sin^2 \gamma \, d\gamma \quad (10)$$

where $\theta = \cos^{-1} \beta$.

For the value of $r = 0.986$, $\theta = \cos^{-1} r = 0.1675$. Equation (10), then, gives us $F(r) = 0.998$. This result suggests that the probability is only $1 - 0.998 = 0.002$ that a correlation coefficient with numerical value as great as 0.986 would have occurred accidentally.

This evidence, with only one sample of four values, is not sufficient to prove the validity of the model proposed. But it seems sufficient to commend further future investigation. Note that, if this model is correct, the slip at $\delta = 58$ mm is more than three times as great as the slip at the tread centerline.

Deduction of Value for Thermal Contact Conductance Between Tire

Tread and Polyimide Surface

The original plan was that contact conductance would be determined directly from figure 13 because the curves for initial rubber temperature for traces 1 and 2 would cross at an abscissa value that could be taken as the correct contact conductance. The tire that produced these two traces had been in contact with the road for only a few revolutions. The two points on the tread surface that touched the two sensors were expected to have been closely at the same temperature before contact. At each of those points, the heat on the tread surface at the end of contact with the road surface would have substantial time to flow deeper into the tread before the next contact. Hence, it was thought

that the two points should have been at temperatures not far apart at the beginning of contact for traces 1 and 2.

Not only did the two curves fail to cross, but it is obvious that, even if they had crossed, they would have done so at an angle so acute that the crossing could not have been depended upon for an accurate determination of contact conductance. The earlier mentioned choice of the lowest supportable value of k_{rs} was made because that value, $k_{rs} = 0.14 \text{ W/(m)(K)}$, brought the two curves closer together than did any higher value of k_{rs} .

Besides the failure of curves for initial rubber temperatures for traces 1 and 2 to cross, a momentarily disturbing observation was that the curves for initial rubber temperature for the four traces appear in almost reverse order to that of the curves for average friction heat. This fact suggests a high negative correlation coefficient between initial rubber temperature and average friction heat.

The correlation coefficients were computed for the various values of contact conductance and are set forth in the following table:

Contact conductance, $\text{W/(m}^2\text{)(K)}$	Correlation coefficient between initial rubber temperature and friction heat
1.2×10^4	-0.84
1.7	-.82
2.6	-.80
3.8	-.77
5.7	-.74

These negative coefficients have considerably higher numerical values than would ordinarily be expected to occur accidentally. However, the accidental occurrence of a negative or positive correlation coefficient of such magnitude would be by no means unique.

The working of the computer program should be expected to produce a false negative correlation if the assumed value of contact conductance were too low. The low contact conductance would hinder the flow of heat to the rubber and the operation of the program would force a finding of an initial rubber temperature too low in order that the rubber could absorb more friction heat. It was even hoped for a time to use this fact as an aid toward deducing the correct value of contact conductance. But the tabulation shows that the negative correlations are affected only slightly by the assumed contact conductance.

No conceivable cause and effect would seem to explain the negative correlation other than too low an assumed contact conductance. Hence, before any further attempt is made to deduce a plausible value of contact conductance from the results it is necessary to examine this negative correlation for its possible reflection on the validity of the computational results. A question naturally arises as to whether a false negative correlation may have been created because of some flaw in the computer program.

Accidental negative correlations may be considered as plausible if we can show that the distributions of initial rubber temperature for the four traces and the distributions of friction heat, as they appear in figure 13, are independently plausible. We have already shown that the friction heat correlates almost perfectly with a reasonable model. Hence, this distribution seems to be independently plausible.

So we will now take up the question of independent plausibility of the computed values of initial rubber temperatures as they appear in figure 13. We cannot formulate a theory that will provide almost perfect correlation with these computed values as we did for the values of friction heat. However, we can show qualitatively that the distribution of the computed values may well be due to other causes than a flaw of the computer program.

Qualitatively, a large negative correlation coefficient may be expected between two sets of variables if the values in the two sets tend to appear in reverse order. In figure 13 we see that the order of descending values of initial rubber temperature is trace 3, trace 1, trace 4, and trace 2. The descending order for average friction heat is trace 4, trace 2, trace 1, and trace 3. Hence, six possible comparisons of order of pairs of values between the two sets are as follows:

Traces in reverse order	Traces in same order
1 and 2	2 and 4
1 and 3	
1 and 4	
2 and 3	
3 and 4	

The five pairs of traces in the left column favor a negative correlation coefficient. The one pair of traces in the right column favor a positive coefficient.

We will first consider traces 1 and 2. After the disappointing failure of the curves to cross became apparent, it soon became clear by simple inspection of figure 2 that the traces indicate (correctly or not) a lower initial rubber temperature for trace 2. The overall drop in indicated temperature of the nickel grid, from the beginning to the end of trace 1, is about 0.068 K. The corresponding temperature drop for trace 2 is about 0.11 K, nearly twice that for trace 1. And the sensor for trace 2 was 62.5 percent farther from the tread centerline than the sensor for trace 1, so that friction heat should

have been greater and the initial rubber temperature should have been hotter, unless there was a substantial differential air cooling.

It seems clear, therefore, that either there was a substantial difference of air cooling or some mismatch existed between the two sensors and their associated equipment, which gave the sensor for trace 2 much greater sensitivity than that for trace 1. The reverse order for these two traces could conceivably be due to one or the other of these conditions rather than to a flaw in the computer program.

The truck, at the rear of which the tire under test was mounted when the records of figures 2 and 3 were obtained, had numerous large appurtenances just in front of the tire. These appurtenances, collectively, almost amounted to a vertical wall in a plane perpendicular to the direction of motion. For this reason, it is possible that the principal organized motion of air relative to the tire was that due to the centrifugal pumping action of the rotating tire and the wheel on which it was mounted. Air flow caused by this pumping action would spiral outward from the axis of rotation along both side walls. Such air flow would cause more rapid cooling of the edges than the centerline of the tread. For each of the two traces recorded by sensor 1 (traces 1 and 3) the sensor was substantially closer to the tread centerline than for the corresponding traces recorded by sensor 2 (traces 2 and 4).

A mismatch between the two sensors and their associated equipment of sufficient magnitude seems most unlikely. Even at the highest value of contact conductance in figure 13, the curves of initial rubber temperature for traces 1 and 2 are separated by about 1.36 K (2.44° F). The curves have become nearly horizontal in this region of contact conductance and it seems they should not approach each other much more closely at higher values of the abscissa for which computations were not run.

Such a difference in air cooling seems much more reasonable than that the difference should be explained by a sensor mismatch. Each element in the tread surface spends more than 10 times as long being cooled by air as it spends in contact with the pavement. As indicated earlier, the computer program only ran to a depth of 2.51×10^{-3} cm (about 9.9×10^{-4} in.) in the rubber because only negligible heat reached that depth during contact with the sensor. Hence, air cooling would only have to extend to that same depth to affect the operation of the computer program.

Hence, it seems plausible that the computed initial rubber temperatures as shown in figure 13 may be correct and that the failure of the curves for traces 1 and 2 to cross is due to differential air cooling.

The same kind of differential air cooling should be expected to make the initial rubber temperature for trace 4 lower than that for trace 3. The distances from tread centerline were 5 mm for trace 3 and 58 mm for trace 4. Except for the differential air cooling, the initial rubber temperature should unquestionably have been higher for trace 4 than for trace 3, rather than lower. The differential air cooling may even have put traces 1 and 4 out of their expected order. The tire for trace 4 had been running on

the pavement under load for 1.6 to 3.2 km while that for trace 1 had been in contact with the pavement for only about 30 m.

Because of the great distance of travel of the tire on the pavement for trace 3 and short distance for traces 1 and 2, and the fact trace 3 at a distance of only 5 mm from tread centerline may have had the least effective air cooling, the orders of traces 1 and 2 relative to trace 3 may be just as they should be.

So it is seen that all the six comparisons shown in the tabulation are either as they were expected to be or may possibly be explained by differential air cooling. The large negative correlations between initial rubber temperature and the friction heat do call for very careful scrutiny of the computer program, which was actually given to the program. But in view of the possibility that has now been explained, that the order of calculated values of initial rubber temperature is correct, the large negative correlations should demand nothing more than the careful scrutiny of the computer program.

It now seems that choosing a valid contact conductance from examination of the curves for initial rubber temperature in figure 13 is just a matter of how much air cooling one is willing to believe. To calculate exactly how much air cooling should occur would be enormously complex and could not be undertaken as part of the program being reported here. At the contact conductance of $1.2 \times 10^4 \text{ W/(m}^2\text{)(K)}$, for trace 2, we would have to assume that the tire surface was cooled by the air to a lower temperature than the 292 K air temperature itself. For trace 4, with a tire tread heated by a substantial distance of running on the pavement, we would have to assume cooling to within 0.5 K of the air temperature. Whether or not appreciable air cooling is believed, such low initial rubber temperatures could not really have existed and no reason is seen to doubt either the accuracy of the records or the validity of the computer program. Hence this value of conductance is clearly unreasonably low even if no appreciable air cooling is assumed. The initial rubber temperatures at the contact conductance of $5.7 \times 10^4 \text{ W/(m}^2\text{)(K)}$ seem most reasonable of any that appear in the curves.

It appears that the curves indicate a contact conductance certainly higher than $1.2 \times 10^4 \text{ W/(m}^2\text{)(K)}$ and possibly as high as or higher than $5.7 \times 10^4 \text{ W/(m}^2\text{)(K)}$. For later use in calculations a value of $3 \times 10^4 \text{ W/(m}^2\text{)(K)}$ will be used. This is a very high conductance for solids in contact. A conductance of this order of magnitude probably exists only because the rubber under pressure deforms to make molecular contact over a large fraction of the supporting surface in spite of asperities on that surface.

It may now be seen that the use of $k_{rs} = 0.14 \text{ W/(m)(K)}$ instead of $k_{rs} = k_r = 0.27 \text{ W/(m)(K)}$ was cautious. For the same assumed contact conductance, if we assume the calculated friction heat unchanged, the higher value of k_{rs} would call for more heat generation on the rubber side of the interface (according to the imaginary model of eq. (1)). Operation of the computer program then would (actually did) call for lower initial rubber temperatures, which would force us to go to higher abscissas in figure 13 to find reasonable initial rubber temperatures. Thus our conclusion about contact con-

ductance would have been higher. But the calculated friction heat would also have been higher. A smaller fraction of this friction heat would have appeared on the polyimide side of the interface. Operation of the computer program would consequently have called for more friction heat in order to explain the temperatures of the nickel grid. So it seems that we would have found higher values of both contact conductance and friction heat with $k_{rs} = k_r = 0.27 \text{ W/(m)(K)}$.

Total Friction Heat and Contribution to Road Resistance

From the cautiously estimated contact intervals for the four temperature traces, the average friction heats shown in figure 13, the vehicle velocity, the tire tread width, the normal load on the tire, and an assumed coefficient of friction, we may easily estimate the total rate of heat generation by friction, the road resistance required to generate that heat, and the average total distance of sliding for all elemental surface areas of the tire relative to the pavement. The vehicle velocity from reference 2 was 22.35 m/sec (50 mph). From the stationary footprint area of 0.0191 m^2 (29.6 in.^2) and length of 0.14 m (5.5 in.) given in reference 2, we get a tread width of 0.137 m. The tire load given in reference 2 was 4448 N (1000 lb). A friction coefficient of 1.0 will be assumed. This may be a high value for sliding friction, but a high value is conservative relative to the average sliding distance that will be calculated. The total rate of heat generation by friction for each trace will be taken from figure 13 at contact conductance of $3 \times 10^4 \text{ W/(m}^2\text{)(K)}$. The results for each trace will be treated as applying over one-quarter of the total tread width. On the basis that has now been described, computational results are as follows:

Trace	Contact interval, msec	Distance of contact, m	Average friction heat, W/m^2	Total rate of heat generation by friction, W
1	7.500	0.168	2.34×10^4	134.6
2	8.000	.179	4.39	260.1
3	8.375	.187	1.61	103.1
4	7.725	.173	5.76	341.2

The distance of contact in column 3 was determined from the contact duration of column 2 and the vehicle speed. The values in the last column are products of average

friction heat and contact area for the specific trace. The total of the values in the last column is the total rate of heat generation, 848 W. The estimated total footprint area from the values in column 3 and the footprint width of 0.137 m is 0.02421 m².

We may now get an approximate value of the road resistance necessary to generate the friction as

$$R_f = \frac{\text{power}}{\text{velocity}} = \frac{848}{22.35} = 37.9 \text{ N (8.52 lb)} \quad (11)$$

The total road surface area contacted per second, the tread width times velocity, is 3.062 m². With the approximation that contact pressure between tire and pavement is uniform, the friction work of 848 J performed in a second must be

$$W_f = A_{rd} p c_f s \quad (12)$$

where A_{rd} is the total road surface area covered by the tire tread per second, 3.062 m², p is the normal pressure, 4448 N (1000 lb) divided by the footprint area of 0.02421 m², c_f is the assumed coefficient of friction, and s is the average distance slipped during contact in meters. The average slip distance required by equation (12) is 1.5 mm (0.06 in.). This average distance slipped is three times as great as was measured in scratch-plate tests reported in reference 2. In those tests small particles of abrasive were between a tire tread and an aluminum plate as the tire was rolled over the plate. The total lengths of scratches in the aluminum plate, caused by the abrasive particles, were measured. This method required an assumption that aluminum particles did not slip, or roll, at all relative to the rubber surface. It would seem one could as well make the assumption that the particles would tend to move more relative to the rubber surface than relative to the aluminum surface. The authors of reference 2 recognized this limitation on the accuracy of their result, and presented the result only as a lower limit of distance slipped.

Both the 37.9-N component of road resistance and the average slip distance of 1.5 mm seem quite reasonable. However, greater confidence in the results will be possible if the 37.9-N road resistance should prove to be compatible with an independent estimate of the component of road resistance required to overcome rubber hysteresis. That independent estimate may be made quite simply with use of thermocouple measurements of temperatures within a tire tread, as will be discussed next.

Heat Generation by Hysteresis Within Tread

Figure 14 is a reproduction of figure 5 of reference 2, modified only by changes of units and omission of information not pertinent to the present discussion. It shows the

temperatures indicated by six thermocouples embedded within the crown of the tire tread at the depths indicated by the circular symbols, and by a single thermocouple attached to the inner surface of the crown. The thermocouple on the inner surface became inoperative after 240 sec and before 480 sec from the start of the run. The thermocouple embedded at a depth of 19 mm (0.75 in.) from the inner surface stopped operating after 720 sec and before 960 sec from the start of the run.

By inspection of figure 14 we may see that, in spite of the loss of two thermocouples, approximate average temperature rises within a time interval may be found by averaging the changes in temperatures indicated by the five thermocouples throughout the test. From the average temperature rise for the entire tread thickness $\overline{\Delta T_i}$ for time interval i , the difference between the average rate of hysteretic heat generation and the average rate of flow of heat out of the tread may be estimated as

$$H_{hi} - H_{oi} \cong A_{tr} d \frac{k_r}{\alpha_r} \frac{\overline{\Delta T_i}}{\Delta t_i} \quad (13)$$

where H_{hi} is the average rate of hysteretic heat generation, H_{oi} is the average rate of flow out of the tread, A_{tr} is the total surface area of the tread (average of inside and outside areas), d is tread thickness, and t_i is the length of the time interval.

Estimation of the rate of flow of heat out of the tread is somewhat hampered by the irregular behavior of the thermocouple at the depth of 19 mm and the loss of the thermocouple on the inner surface. Early in the test the thermocouple at 19-mm depth showed a more rapid temperature rise than any other, but later in the test it showed a slower rise. Apparently we may take its erratic behavior as being an indication of its impending failure and disregard all of its indications. For the records at 40, 100, 180, and 240 seconds, we may observe that the thermocouple at the inner surface and the two thermocouples at depths of 7 and 9 mm showed temperatures approximately linear with depth. An assumption will be made here that such condition would have continued if the surface thermocouple had remained operative.

On such a basis, estimates may be made of the numerical value of the slope of each temperature curve in figure 14 for each time j after the start of the test at the inner surface, m_{1j} , and at the outer surface, m_{2j} . From these numerical values of slopes, the flow of heat out of the tread may be estimated as

$$H_{oj} \cong k_r (A_1 m_{1j} + A_2 m_{2j}) \quad (14)$$

where A_1 is the inner surface area of the tread and A_2 is the outer surface area. For the nine temperature curves in the figure ($j = 1$ to 9), the application of equation (14) yields values shown in table I for H_{oj} with use of a total tread inner surface area

$A_1 = 2760 \text{ cm}^2$, and total tread outer surface area $A_2 = 2960 \text{ cm}^2$.

For use in equation (13), we may now use from table I the average of the two values of H_{oj} at the beginning and end of the i^{th} time interval. That is, if $i = 1$ corresponds to the time interval between the curves for $j = 0$ and $j = 1$, $i = 2$ corresponds to the interval between the curves for $j = 1$ and $j = 2$, and so on, then, for use in equation (13),

$$H_{oi} = \frac{1}{2} \sum_{j=i-1}^i H_{oj} \quad (15)$$

Accordingly, with use of equations (13) and (15) and table I, approximate rates of hysteretic heat generation H_{hi} for the median times of the various time intervals i were calculated and listed in table II. For the calculations the median value of tread area was $A = 2858 \text{ cm}^2$. The tread thickness was 23 mm. Also entered in table II are the forces that must exist as drags on the motion of the wheel at the test speed 22.4 m/sec, to provide the energy consumed by the tread hysteresis.

The hysteresis-induced drag forces shown in table II are plotted relative to time from start of test in figure 15. A smooth curve in the figure approximately represents the results. The drag force due to tread hysteresis is apparently three to four times as great for the cold tire as it is for the tire that has approached equilibrium thermal conditions after about 1000 sec of running at the test speed. Presumably a similar change in drag due to sidewall hysteresis would also occur.

In reference 2, a hysteresis-induced drag of about 89 N (~20 lb) was estimated on a thermal basis for the entire tire. This determination, however, applied to the time interval from 40 to 60 sec after the start of the test. Under the assumption that the reduction of hysteresis in the sidewalls would be proportional to that of the tread, and by comparison of the 40- to 60-sec region in figure 15 to the region beyond 1000 sec, we see that the 89-N estimate in reference 2 should be reduced to about 21.6 N (about 5 lb) for the tire under the much hotter thermal equilibrium conditions.

Traces 3 and 4 of figure 3 were recorded after the tire had been run for a distance not accurately measured or recorded but thought to be of the order of 1.61 km (1 mile). The time of recording of the traces would therefore very roughly correspond to $i = 2$ in table II. We may only speculate that the same component of drag force of 37.9 N to overcome friction forces (according to eq. (11)) would apply after the tire reached thermal equilibrium. However, it appears that the 1.5-mm average slide of an element of rubber surface must be due to the geometry involved in producing a flat area moving steadily around the periphery of a toroid. We would expect that tremendous internal stresses would be required in the rubber to change the slide pattern or magnitude substantially. The same component of road resistance of about 37.9 N should therefore

probably be assumed tentatively to exist after thermal equilibrium is reached. Under that assumption, after establishment of thermal equilibrium, the total drag of the tire should be approximately the 37.9 N due to friction involved in slippage plus the estimate from reference 2 for hysteresis as adjusted to 21.6 N, or a total of about 59.5 N (13.4 lb), of which about 64 percent is required to overcome friction between tire and pavement and about 36 percent to overcome rubber hysteresis.

A certain discrepancy still needs to be considered. By double interpolation in table I, at an elapsed time of about 70 sec, the rate of flow of heat through the outer surface of the tread must be about 28 W, or about 1160 W/m^2 . With the contact conductance earlier derived, of the order of $3 \times 10^4 \text{ W/(m}^2\text{)(K)}$, which would be even higher between rubber and pavement, the flow of 1160 W/m^2 would require an average temperature difference during contact with the pavement of only a very small fraction of a degree Kelvin. And yet, the trends at 40 and 100 sec in figure 14 suggest that the rubber surface temperature at about 70 sec should have been perhaps 7 K above the road surface temperature. In figure 13, from the appearance of the curve (straight line) of initial rubber temperature for trace 3, it is apparent that the temperature of the rubber surface must have been about 0.8 K relative to the polyimide (or road surface) temperature when the two first came into contact. Such a condition would be necessary in order that the computed initial rubber temperature should be independent of assumed contact conductance. But the tire had been rolling on the road for a time interval of the order of 70 sec.

The discrepancy may possibly be explained by the following considerations:

- (1) The distance run before the making of traces 3 and 4 (ref. 2) was not actually measured. The time could well have been substantially less than the 70 sec estimated.
- (2) As the air temperature was about 5 K less than the road temperature, the temperature of the tire at the start of the run could well have been substantially less than that of the road, so that the temperature reached after about 70 sec might have been considerably less relative to the road than relative to the initial temperature of the tire.
- (3) The temperature of about 302 K (84° F) measured for the tire with a contact pyrometer had to be taken not only after the making of traces 3 and 4 but also after an unspecified amount of additional running, which may have heated the tire appreciably hotter. Because of these possible explanations of the discrepancy, it is not believed that it should overbalance the weight of this analysis.

SUMMARY OF RESULTS OF THERMAL ANALYSIS

The thermal analysis has indicated the following:

- (1) Contact thermal conductance between tread surface and polyimide surface is higher than 1.2×10^4 and may be higher than $5.7 \times 10^4 \text{ W/(m}^2\text{)(K)}$. A value of $3 \times 10^4 \text{ W/(m}^2\text{)(K)}$ seems reasonable for use.

(2) The distance slipped between an element of rubber surface and pavement varies as the square root of the sum of two components, (a) a constant and (b) the product of another constant by the square of the distance of the element from the tread centerline.

(3) For the size and type of free rolling tire used in the experiments, the average distance of slip between any element of the tire surface and the road surface due to squirming within the contact area is about 1.5 mm. This slip occurs as a complicated series of stick and slip conditions.

(4) The friction work involved in the slippage due to squirming causes a tire drag of about 37.9 N (8.5 lb), or about 64 percent of the total drag of a tire that has reached thermal equilibrium.

(5) Although the part of the tire drag caused by mechanical hysteresis within the rubber is predominant when the tire is cold, it gradually decreases by 75 percent or more as the tire heats, so that for the tire that has reached thermal equilibrium mechanical hysteresis accounts for only about 36 percent of the total tire drag.

APPLICATION OF RESULTS TO ALUMINUM SURFACED RUNWAY

In reference 1, a number of concepts were discussed by which aircraft braking might possibly be improved by providing a highly conductive surface. Great difficulty has always been encountered in getting good wet friction between a tire and any metal surface, including aluminum. Yet, as illustrated and discussed in reference 1, very good friction coefficients can easily be obtained between small pieces of rubber and a moderately clean aluminum surface, even with a stream of water flowing on it. Water does not even wet the moderately clean aluminum surface. Before the use of an aluminum surface can be seriously considered it seems necessary that some basic research be done to determine just why the difficulty exists with a tire under load, even at very low speeds. Finding the reason might result in a solution. That solution might possibly involve a grooving or perforation pattern of the aluminum surface or some provision for porous drainage of the surface. In the meantime, pending such basic research, other concepts could be considered. The concept listed as most probable of success in reference 1 was attainment of a substantially greater braking force than is now possible, perhaps as great as 1.4 times the vertical load, with a textured runway surface that exposes tread rubber to small areas of aluminum and small areas of abrasive material. This concept could not have been quantitatively evaluated because both the rate of generation of heat by friction and the thermal contact conductance were unknown. To make possible a quantitative appraisal of that concept was one of the principal incentives for conduction of the analysis that has been described here.

The possibility now exists for such a quantitative appraisal. However, only a superficial examination of the problem will be presented here for the purpose of illus-

trating the potentiality.

We shall seek an estimate of the maximum temperature reached within the footprint of a free rolling aircraft tire because of friction due to slippage associated with squirming. We will then consider whether that temperature is great enough to cause softening of the rubber surface and consequent degradation of friction coefficient.

An aircraft tire will be considered with an inflation pressure of about 1.38×10^6 N/m² (200 psi) and with a footprint $2\frac{1}{2}$ times as long as that of the tire with which the traces in figures 2 and 3 were obtained. The average footprint length determined from the four temperature traces for the automotive tire was 0.177 m. Thus the assumed footprint length for the aircraft tire will be 0.442 m. The pressure between tread surface and runway will be assumed uniform and equal to the inflation pressure. A landing speed of 82.3 m/sec (160 knots) will be assumed. Coefficient of friction will be taken as 1.0, as with the automotive tire. The average distance of slip will be assumed as 3.75 mm, $2\frac{1}{2}$ times that found for the automobile tire. Because the dimensions of the flat spot on the aircraft tire are greater relative to the circumference of the tire, it is believed the $2\frac{1}{2}$ -fold assumption of average sliding distance is conservative.

As the footprint of the aircraft tire is approximately elliptical, the average time of contact of an elemental area of rubber surface with the runway is appreciably less than the centerline length of the footprint. Regardless of the width of the elliptical footprint, it can easily be shown that the average length on an area basis is $8/3\pi$, or about 0.85, times the centerline length. Thus, with a velocity of 82.3 m/sec and with the centerline length of 0.442 m, the average time spent within the footprint by an elemental area of rubber is about 4.56×10^{-3} sec. The average rate of heat generation by friction, then, must be the product of the normal pressure, the coefficient of friction, and the average length of slip, divided by the average time of contact, or

$$g(t) = \frac{1.38 \times 10^6 \times 1.0 \times 3.75 \times 10^{-3}}{4.56 \times 10^{-3}} \text{ W/m}^2 = 1.135 \times 10^6 \text{ W/m}^2 \quad (16)$$

For application of equation (1) or (A7), with use of the conductivity of asphalt instead of that of polyimide, we have

$$\frac{k_{rs}}{k_{rs} + k_{as}} g(t) = \frac{0.14 \times 1.135 \times 10^6}{0.14 + 0.111} \text{ W/m}^2 = 6.33 \times 10^5 \text{ W/m}^2 \quad (17)$$

and

$$\frac{k_{as}}{k_{rs} + k_{as}} g(t) = \frac{0.111 \times 1.135 \times 10^6}{0.14 + 0.111} \text{ W/m}^2 = 5.02 \times 10^5 \text{ W/m}^2 \quad (18)$$

Now $6.33 \times 10^5 \text{ W/m}^2$ will flow into the rubber with a slower rise of surface temperature than will exist with $5.02 \times 10^5 \text{ W/m}^2$ flowing into the asphalt. However, the contact conductance found in this analysis is so high that no very great temperature difference should be needed between the two surfaces to keep their temperatures increasing at about the same rate. So we may get a useful result if we determine the distribution of the friction heat between the two surfaces that will cause their temperatures to increase at the same rate. To get the same temperature on both surfaces at the end of any time interval during contact, we may equate right sides of equation (3) for rubber and asphalt and solve for φ_r/φ_{as} , getting

$$\frac{\varphi_r}{\varphi_{as}} = \frac{k_r}{k_{as}} \left(\frac{\alpha_{as}}{\alpha_r} \right)^{1/2} = 1.811 \quad (19)$$

where φ_r and φ_{as} are fluxes into rubber and asphalt, respectively. Such ratio, with a total $g(t)$ of $1.135 \times 10^6 \text{ W/m}^2$, calls for a flow of heat into the rubber at a rate of $7.31 \times 10^5 \text{ W/m}^2$ and into the asphalt at $4.03 \times 10^5 \text{ W/m}^2$. If we use the equivalent imaginary condition represented by equation (1) (see eqs. (17) and (18)), this result means that $9.8 \times 10^4 \text{ W/m}^2$ will flow through the contact conductance from asphalt to rubber in order to keep the surface temperatures equal.

In reference 1, it is shown theoretically that contact thermal conductance between two materials such as rubber and polyimide may be expressed approximately as

$$h = \left(h_{rs}^{-1} + h_{pol}^{-1} \right)^{-1} = \frac{h_{pol} h_{rs}}{h_{rs} + h_{pol}} \quad (20)$$

where h_{rs}^{-1} and h_{pol}^{-1} are contact resistances within the rubber and polyimide surfaces and that

$$\frac{h_{rs}}{h_{pol}} = \frac{k_{rs}}{k_{pol}} \quad (21)$$

It is also argued there that, if a different material is substituted, such as asphalt for polyimide, with similar contact conditions, we may substitute h_{as} as defined by the following equation for h_{pol} in equation (20):

$$h_{as} = \frac{k_{as}}{k_{pol}} h_{pol} \quad (22)$$

By solving equations (20) and (21) for h_{rs} and h_{pol} , then calculating a value of h_{as} with equation (22), and finally substituting the value of h_{as} for h_{pol} in equation (20), with use of a contact conductance of $3 \times 10^4 \text{ W/(m}^2\text{)(K)}$ for the rubber-polyimide interface, $k_{pol} = 0.156 \text{ W/(m)(K)}$, $k_{as} = 0.111 \text{ W/(m)(K)}$, and $k_{rs} = 0.14 \text{ W/(m)(K)}$, we derive a contact conductance for the rubber-asphalt interface of $2.52 \times 10^4 \text{ W/(m}^2\text{)(K)}$. With such a contact conductance, the flow of $9.8 \times 10^4 \text{ W/m}^2$ from asphalt surface to rubber surface would require a temperature difference of only about 3.89 K. On later contacts of the tire with other positions on the asphalt, the temperature of the tire surface would be initially higher and the mean temperature difference between the surfaces would reduce or even reverse.

We now see that we will get very nearly the maximum temperature elevation of the rubber surface if we use equation (3) for computing the temperature of the asphalt surface, using the thermal flux of $5.02 \times 10^5 \text{ W/m}^2$ from equation (18) that is generated by friction on the asphalt surface minus the liberal estimate of $9.8 \times 10^4 \text{ W/m}^2$ flowing from asphalt to rubber. Thus we get a net flow of $4.04 \times 10^5 \text{ W/m}^2$ into the asphalt. The result, for the average interval of $4.56 \times 10^{-3} \text{ sec}$, is an increase of temperature of the rubber surface by 68.8 K. If we assume an initial runway temperature of 311 K (100° F), we will have rubber surface temperatures of about 380 K (224° F).

This temperature might not be sufficient to soften the rubber surface significantly, and thereby reduce the friction coefficient, but we observe that it exists without application of any braking force. Under conditions of braking, it is necessary to increase the total part of the area within the footprint where forward slip occurs and to reduce the total part where backward slip occurs. An increase in maximum distance slipped results.

Also, this calculation was made for average conditions, including average distance slipped. As we saw earlier, the distance slipped varies greatly with distance from the tread centerline and the maximum distance slipped is therefore substantially greater than the average with consequent greater local heating.

We do not have the necessary data to calculate the maximum temperatures that might be reached, for maximum slip under braking conditions, but it should be substantially greater than the 380 K calculated previously.

Softening of the rubber because of high local temperatures, even for very short times, might cause viscous shear of the rubber which would generate heat under the rubber surface from whence it could not flow rapidly either to the tire surface or deeper into the tread. Temperatures within the shearing region close to the surface would then go far higher than the values estimated previously.

Now if we rearrange equation (3), we have

$$u(0, t) = 2\varphi \left(\frac{t}{k\rho c\pi} \right)^{1/2} \quad (23)$$

where ρ is the density and c the specific heat of the material. We see that, if we increased the $k\rho c$ product by even as little as twofold, for example, by adding aluminum particles to the aggregate used for the surface layer of the runway, we would reduce the average temperature elevation of 68.8 K to about 48.6 K giving us an average temperature leaving contact of 359.6 K (about 188° F) instead of 380 K (225° F).

It is appreciated that the results described are a greatly compounded extension from the raw data seen in figures 2 and 3 and that they could therefore be only approximate. But even with a wide margin of possible error, they strongly suggest that braking could probably be improved by even a moderate increase in heat absorbing capacity of the runway surface.

CONCLUDING REMARKS

The analysis reported herein was conducted with use of minimal experimental data. For that reason conclusions can be of only an approximate, or tentative, nature. It is believed the results are contrary to existing suppositions to such an extent that they should justify further research into the questions of how much heat is generated by friction between tread and asphalt for a free rolling tire, how such heat generation may be reduced, and how the generated heat is dissipated.

With due regard to the need for more extensive research, the following tentative conclusions seem to be justified:

1. In the free rolling of a tire on a hard surface, a substantial amount of slipping or squirming (in a stick-slip manner) occurs between elementary tread surfaces and the supporting surface (approximately an average of 1.5 mm in the tests analyzed here).

2. Slippage of elements of rubber surface relative to pavement, for a free rolling automotive tire, at least approximately conforms to the concept that a longitudinal slip of constant magnitude occurs for all distances from the tread centerline, simultaneously with a transverse slip whose magnitude is proportional to the distance from the tread centerline.

3. Friction between the tread of a free rolling tire and the supporting surface, due to slippage, comprises a significant fraction of the road resistance.

4. The same type of friction must account for a substantial fraction of tread wear.

5. The thermal contact conductance between tire tread and supporting surface is not low enough to provide a substantial barrier to the transfer of heat.

6. Friction generated by the squirming between a runway surface and the tread of an aircraft tire that is not really skidding may develop enough heat to limit the braking force at which skidding begins.

7. Moderate increase of the product of conductivity, density, and specific heat, for a thin layer on the surface of a runway, would result in substantially lower surface temperatures, thereby permitting increases in maximum available braking force.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 28, 1975,
505-08.

APPENDIX A

FINITE-DIFFERENCE METHOD FOR THERMAL ANALYSIS

A type of Crank-Nicholson scheme (ref. 9) was used for the numerical analysis of heat flow on the surface of and within both the nickel-grid sensors and the tire tread. Various entities that will be used in description of the analytical method are identified in figure 4.

Equation (1. 9) in reference 9 for one-dimensional heat flow reads as follows:

$$\frac{u_{j, n+1} - u_{j, n}}{\Delta t} = \alpha \frac{u_{j+1, n} - 2u_{j, n} + u_{j-1, n}}{(\Delta x)^2} \quad (A1)$$

where $u_{j, n}$ is temperature of midpoint of layer j at time n , Δt is interval between time n and time $n + 1$, α is thermal diffusivity, and Δx is thickness of a layer. Equation (1. 21) of the same reference, with similar notations, reads as follows:

$$\frac{u_{j, n+1} - u_{j, n}}{\Delta t} = \alpha \frac{u_{j+1, n+1} - 2u_{j, n+1} + u_{j-1, n+1}}{(\Delta x)^2} \quad (A2)$$

The first step in developing the basic finite-difference equation for use in the present analysis was to write an equation with the left side the same as that of equation (A1) or (A2), but with the right side a simple arithmetical average of the right sides of equations (A1) and (A2), giving,

$$\frac{u_{j, n+1} - u_{j, n}}{\Delta t} = \frac{\alpha}{2} \frac{u_{j+1, n} + u_{j+1, n+1} - 2u_{j, n} - 2u_{j, n+1} + u_{j-1, n} + u_{j-1, n+1}}{(\Delta x)^2} \quad (A3)$$

Equation (A3) in that form is suitable for application to a semi-infinite plate of one uniform material, divided into layers of uniform thickness. It was desired here, particularly for application to the nickel-grid sensor, to be able to apply the equation to layers of differing thickness and/or material. Accordingly, equation (A3) was revised to read,

$$\begin{aligned} \frac{c\rho(\Delta x)_j}{\Delta t} (u_{j,n+1} - u_{j,n}) = & \frac{1}{2} K_{j,j+1} (u_{j+1,n+1} + u_{j+1,n} - u_{j,n+1} - u_{j,n}) \\ & - \frac{1}{2} K_{j-1,j} (u_{j,n+1} + u_{j,n} - u_{j-1,n+1} - u_{j-1,n}) \end{aligned} \quad (A4)$$

where c is the specific heat of the material in layer j , ρ is the density of the material in layer j , $(\Delta x)_j$ is the thickness of layer j , and where

$$K_{j,j+1} = \left[\frac{(\Delta x)_j}{2k_j} + \frac{(\Delta x)_{j+1}}{2k_{j+1}} + h_{j,j+1}^{-1} \right]^{-1} \quad (A5)$$

In equation (A5), k_j is the thermal conductivity of the material in layer j , and $h_{j,j+1}$ is the contact conductance of the interface between layers j and $j+1$ (always to be taken as infinite when the layers j and $j+1$ are contiguous parts of a single solid material).

When all values $u_{1,n}$, $u_{2,n}$, $u_{3,n}$, \dots , have been determined, the only unknowns in equation (A4) are $u_{j,n+1}$, $u_{j+1,n+1}$, and $u_{j-1,n+1}$. Hence, simultaneous equations for $u_{1,n}$, $u_{2,n}$, $u_{3,n}$, \dots , up to the greatest value of j that is significant, form an easily solved tridiagonal matrix for the temperatures $u_{1,n+1}$, $u_{2,n+1}$, $u_{3,n+1}$, \dots .

In application of equation (A4) to the nickel-grid sensor, a tentative history of thermal flux through the upper surface was always used. That flux, for the interval from time n to time $n+1$, was substituted for the second term in the right side of equation (A4) for $j=1$. The maximum value of j , j_{\max} , was made as great as necessary in order that $u_{j_{\max},n}$ for any n would not be significantly greater or less than the initial value $u_{j_{\max},0}$. Then the first term on the right side of equation (A4) was omitted for $j=j_{\max}$. Finally, after solution of the system of equations, the surface temperature of the sensor was determined as

$$u_{0,n+1} = u_{1,n+1} + \frac{1}{2} \varphi_{p(n,n+1)} \frac{(\Delta x)_1}{k_1} \quad (A6)$$

where $\varphi_{p(n,n+1)}$ is the average thermal flux through the upper surface of the polyimide for the interval from time n to time $n+1$.

In application of equation (A4) to the tire tread, given an assumed contact conductance between rubber and polyimide and given a history of temperatures on and thermal fluxes through the polyimide surface, a slight complication was of course involved for the first layer. Here, the thermal flux through the outer tread surface during the in-

terval from time point n to time point $n + 1$ is composed of two parts, according to equation (1). One of those parts $\varphi_{c(n, n+1)}$ is conductive flux through the contact conductance from the polyimide surface; the other $\varphi_{rf(n, n+1)}$ is friction heat generated on the rubber surface. Neither $\varphi_{c(n, n+1)}$ nor $\varphi_{rf(n, n+1)}$ is known. However, we may use results from the thermal analysis for the nickel-grid sensor, namely, $\varphi_{p(n, n+1)}$ and the average temperature of the polyimide surface during the interval from time point n to time point $n + 1$, $u_{0p(n, n+1)}$.

Implicit in equation (1) is the following equation:

$$-k_{pol} \frac{\partial u_{pol}(x, t)}{\partial x} = \left(\frac{k_{pol}}{k_{pol} + k_{rs}} \right) g(t) + h [u_r(x, t) - u_{pol}(x, t)], \quad x = 0 \quad (A7)$$

A finite-difference analogue of that equation is

$$\varphi_{p(n, n+1)} = \left(\frac{k_{pol}}{k_{pol} + k_{rs}} \right) \varphi_{f(n, n+1)} + h [u_{0r(n, n+1)} - u_{0p(n, n+1)}] \quad (A8)$$

where $\varphi_{f(n, n+1)}$ is the average rate of heat generation by friction on both sides of the interface between polyimide and rubber during the interval from time point n to time point $n + 1$, and $u_{0r(n, n+1)}$ and $u_{0p(n, n+1)}$ are the average temperatures on rubber surface and polyimide surface respectively during the same interval. The only unknowns in equation (A8) are $\varphi_{f(n, n+1)}$ and $u_{0r(n, n+1)}$. We may eliminate $g(t)$ from equations (1) and (A7), giving

$$-k_{rs} \frac{\partial u_r(x, t)}{\partial x} = -k_{rs} \frac{\partial u_{pol}(x, t)}{\partial x} + h \frac{k_{rs} + k_{pol}}{k_{pol}} [u_{pol}(x, t) - u_r(x, t)], \quad x = 0 \quad (A9)$$

A finite-difference analogue of equation (A9) is

$$\varphi_{r(n, n+1)} = \frac{k_{rs}}{k_{pol}} \varphi_{p(n, n+1)} + h \frac{k_{rs} + k_{pol}}{k_{pol}} [u_{0p(n, n+1)} - u_{0r(n, n+1)}] \quad (A10)$$

where $\varphi_{r(n, n+1)}$ is average rate of flow of heat into rubber surface during the interval from time point n to time point $n + 1$. Now, for $j = 1$, we may substitute the positive left side of equation (A10) for the negative second term in the right side of equation (A4), giving

$$\begin{aligned} \frac{c\rho(\Delta x)_1}{\Delta t} (u_{1,n+1} - u_{1,n}) = & \frac{1}{2} K_{1,2} (u_{2,n+1} + u_{2,n} - u_{1,n+1} - u_{1,n}) + \frac{k_{rs}}{k_{pol}} \varphi_{p(n,n+1)} \\ & + h \frac{k_{rs} + k_{pol}}{k_{pol}} [u_{0p(n,n+1)} - u_{0r(n,n+1)}] \end{aligned} \quad (A11)$$

Equation (A11) has three unknowns, namely, $u_{0r(n,n+1)}$, $u_{1,n+1}$, and $u_{2,n+1}$. This equation, then, is suitable for the second row in our tridiagonal matrix. But we need an equation for the first row of the matrix containing only $u_{0r(n,n+1)}$ and $u_{1,n+1}$ as unknowns. Such an equation may be easily obtained by substituting

$\left[u_{0r(n,n+1)} - \frac{1}{2} (u_{1,n} + u_{1,n+1}) \right] \frac{2k_r}{(\Delta x)_1}$ for its approximate equivalent $\varphi_{r(n,n+1)}$ in equation (A10), giving

$$\begin{aligned} \left[2u_{0r(n,n+1)} - u_{1,n} - u_{1,n+1} \right] \frac{k_r}{(\Delta x)_1} = & \frac{k_{rs}}{k_{pol}} \varphi_{p(n,n+1)} \\ & + h \frac{k_{rs} + k_{pol}}{k_{pol}} [u_{0p(n,n+1)} - u_{0r(n,n+1)}] \end{aligned} \quad (A12)$$

(Here we use k_r instead of k_{rs} because k_{rs} applies only as far down as the depth of surface asperities. The depth to which k_r applies in eq. (A12) amounts to one half the thickness of the uppermost layer of rubber.) So equation (A12) for the first row, equation (A11) for the second row, and equation (A4) for subsequent rows, make up a tridiagonal matrix that can readily be solved for $u_{j,n+1}$ for $j \geq 1$ and for $u_{0r(n,n+1)}$. Equation (A10) may then be solved for $\varphi_{r(n,n+1)}$. Finally, equation (1) may be solved for $g(t)$.

APPENDIX B

RECONCILIATION OF RECENT RESEARCH AT UNIVERSITY OF MICHIGAN WITH THIS REPORT

Reference 3 came to the author's attention after this report was written and after his retirement, but before publication of this report. Rather than undertake substantial revision of the body of this report to include analysis and discussion of reference 3 wherever pertinent, it was decided to restrict such analysis and discussion to this appendix. Such analysis and discussion will begin with the question of depth at which heat is generated by friction.

NATURE OF GENERATION OF HEAT BY FRICTION

A part of reference 3 that is vital to the present report is Section VI, entitled GENERATION OF HEAT DUE TO SURFACE EFFECTS. This section of reference 3 is directed toward a theoretical estimation of the effective depth beneath a rubbing surface at which friction heat is generated and toward the question of the fraction of the total friction heat generated within each of two different materials when their surfaces are rubbed together.

Equation (1) was derived in reference 1 under the theory that the heat of friction is generated within the surface layers of atoms or, at most, just a few atomic layers beneath each surface. If, instead, the friction heat were generated at a substantial depth beneath either surface, then equation (1) would not be valid and the entire analysis reported here would have to be carefully re-examined as to whether its results are significant.

Theories about the cause of friction and the manner of generation of heat by friction are still controversial. Probably the oldest theory that still has its adherents is that of Coulomb. According to Coulomb's theory, mutual adhesion of the rubbing surfaces plays only a minor role, if any, in friction. Instead, this theory holds that frictional force is caused by the action of myriads of asperities on the two surfaces climbing up on each other like pairs of mating wedges. Such wedging action would require an equilibrium between force vectors normal to the gross surfaces and force vectors tangential to those surfaces. According to this theory, the net effect of these many minute tangential vectors constitute the frictional force.

According to this theory, generation of heat would be due to hysteresis involved in the repeated deformation and relaxation of material at substantial depths beneath the

surface, caused by the localized wedging action. The derivation of equation (1) (eq. (B12) in ref. 1) would not be valid.

According to more modern theory, asperities have little effect except to account for the fact that the effective rubbing area varies as the normal load between two metals, or as a power of the normal load somewhat less than unity when a polymer is involved. According to this theory, as clearly explained recently by Merchant (ref. 10), the frictional force is caused mainly by adhesion between surface atoms of the two rubbing materials, though some frictional force may be caused by asperities on the harder surface "plowing" through the softer surface. The atomic adhesion does not require the presence of asperities, except that the individual adhering atoms themselves might be regarded as asperities.

Defense of the Coulomb theory was presented by Bikerman in his discussion of Merchant's report (ref. 10). In that discussion Bikerman freely acknowledged that the adhesion theory represents the majority view, as did Ku in his later nonpartisan discussion.

The present author can add little indeed to the discussions among Merchant, Bikerman, Ku, Tabor, and others when he states that he himself adheres to the adhesion theory and consequently believes that his equation (eq. (1)) is approximately correct.

Clark and Staples in reference 3 report about the same coefficient of friction (1.4) for a rubber tire on a smooth dry aluminum surface as on a dry concrete or asphalt surface. Such a comparison seems to suggest little effect indeed from "plowing" of asperities through the rubber. If the "plowing" contribution to friction were substantial, we should expect a much greater coefficient with concrete or asphalt than with smooth aluminum.

In figure 16 a single asperity on a hard supporting surface is shown for simplicity as a right prism of isosceles triangular cross section whose axis is perpendicular to the plane of the figure and extends at right angles to the direction of slide of the rubber surface above it. Adhesion between rubber and solid material is assumed nonexistent, so that the only forces exerted by the prismatic asperity on the rubber are P_{n1} and P_{n2} , each normal to a surface of the prism. These two forces are resolved into components P_1 and F_1 on the uphill side of the prism and components P_2 and F_2 on the downhill side of the prism.

Because of the hysteresis loss factor η used by Clark and Staples, we may write

$$\left. \begin{aligned} P_{n2} &= (1 - \eta)P_{n1} \\ P_2 &= (1 - \eta)P_1 \\ F_2 &= (1 - \eta)F_1 \end{aligned} \right\} \quad (B1)$$

The net frictional force exerted by the asperity against the motion of the rubber will be

$$F = F_1 - F_2 \quad (B2)$$

and the total upward reaction of the asperity against the load will be

$$P = P_1 + P_2 \quad (B3)$$

The tangent of each of the angles β shown in figure 16 will be

$$\tan \beta = \frac{F_1}{P_1} = \frac{F_2}{P_2} \quad (B4)$$

The coefficient of friction must be

$$\mu = \frac{F}{P} \quad (B5)$$

Solving equations (B1) to (B5), we get

$$\tan \beta = \frac{2 - \eta}{\eta} \mu \quad (B6)$$

Note that the value of β given by equation (B6) must exist for the average asperity if the sliding friction coefficient μ is to exist. If we use the coefficient 1.4 found by Clark and Staples and a value $\eta = 1/2$ which is probably unreasonably large, we get about 76.5° for the value of β . Even with $\eta = 0.9$, $\beta = 59.7^\circ$. Williamson (ref. 11) found that typical asperities on surfaces of solids have slopes occasionally as steep as 25° but that the slopes are usually between 5° and 10° .

It appears, therefore, that the Coulomb concept of friction is quite untenable for the rubber tire sliding on a smooth aluminum surface.

The analysis by Clark and Staples on generation of heat by friction may have an approximate application to the adhesion concept of friction. In considering such application we will, for the present, ignore the effect on heat generation due to the rupture of rubber-to-rubber bonds. As many rubber-to-aluminum bonds would be created exothermically as would be ruptured endothermically. The ruptures of rubber-to-rubber bonds would be extremely close to the rubber-aluminum interface. If, on the average, such ruptures occurred at depths of more than a few atomic diameters beneath the tread surfaces, treads would not last as long as they do. Moreover, rupture of rubber-to-

rubber bonds must be endothermic.

The treatment by Clark and Staples ignores forces normal to the rubbing surfaces and apparently does not need to take those forces into account for its purposes. Mathematical expressions are developed for stresses and strains within a semi-infinite solid whose surface is infinitely smooth. The stresses and strains are treated as being caused by intermittent (stick-slip) movement of a minute asperity of the other body along the infinitely smooth surface. That is, a model is used according to which a frictional force is intermittently applied, tangentially to the infinitely smooth surface, along a line that lies in the surface at right angles to the direction of the force.

Such a model corresponds with the atomic interactions presumed to generate frictional heat by Tabor in one of his responses to Bikerman in the discussion of Merchant's report (ref. 10). It is true that an atom displaced by adhesion does not correspond to the line application of a force assumed by Clark and Staples, but a line of adhering atoms within the surface could be considered without undue violence to the adhesion concept of friction.

According to this model, Clark and Staples developed the following equation for differential heat generation for a single cycle of application of the force:

$$dQ = \frac{P^2 \eta}{4\pi E} \cdot \frac{dx}{x} \quad (B7)$$

where P is the tangential force, E is modulus of elasticity, and x is distance beneath the surface.

The definite integral of equation (B7), for total heat generation occurring at all depths between x_1 and x_2 is

$$Q(x_1 < x < x_2) = \frac{P^2 \eta}{4\pi E} \ln\left(\frac{x_2}{x_1}\right) \quad (B8)$$

This equation has the obvious faults that $Q(x_1 < x < x_2)$ approaches infinity as x_2 approaches infinity or as x_1 approaches zero. Hence, the range of values of x throughout which the equation may be applied is subject to judgment and the practical value of the equation is consequently somewhat limited.

A related limitation of the usefulness of equation (B8) as applied to a stick-slip phenomenon is due to the fact that its derivation involved the implicit assumption that the force P is applied long enough that a steady-state distribution of strains and stresses is achieved. For such a steady-state distribution at an infinite depth x_2 the force P would have to be applied for infinite time. Clearly, equation (B8) could not be used with a greater value of x_2 than the depth to which a transverse wave could travel during one cycle of application of the force P .

The adhesion of atoms at the surfaces of contact for extremely short intervals of time could cause an approximation of the steady-state distribution of stresses and strains only at very shallow depths. An exact solution of the problem dynamically would require a major mathematical effort, which cannot be undertaken here. The solution would have to involve treatment of an infinite series of transverse waves traveling from the surface into the depths of the rubber, with due regard to the attenuation of each wave at every depth because of mechanical hysteresis. However, a useful approximate solution seems to be possible with use of equation (B8) if we use for x_2 the depth to which a transverse wave could travel during one cycle of application of force. Neglect of heat generated at depths greater than x_2 because of the eventual travel of transverse waves to greater depths should not be too serious because of the very large value of η for tread rubber. Also, such neglect would be countered by the fact that stresses and strains resulting from transient surface forces would tend to be concentrated at positions nearer the surface than called for by equation (B7).

We may gain some conception of the ease or difficulty involved in the flow of heat to the rubbing surface, from the depths at which it is generated according to equation (B7), by considering two imaginary conditions as follows: (1) the heat is generated according to equation (B7), but is all compelled to flow to the surface because of a plane parallel to the rubbing surface and having zero conductance, at the depth x_2 ; and (2) the same quantity of heat is generated at an effective depth x_e , and it is all compelled to flow to the surface by a plane parallel to the rubbing surface and having zero conductance, at a depth infinitesimally deeper than x_e . For the purpose of the comparison, we will assume that the temperature reached at depth x_e for condition (2) is equal to the average temperature reached within the region between x_1 and x_2 for condition (1).

For the imaginary condition (1), if we assume that enough cycles of heating according to equation (B8) have occurred for a virtual steady-state temperature distribution to exist, then

$$\frac{du}{dx} = \frac{f}{k} \left(Q - \frac{P^2 \eta}{4\pi E} \right) \ln \left(\frac{x}{x_1} \right) \quad (B9)$$

where u is the temperature at distance x below the surface and f is the frequency of the stick-slip cycle. From equation (B9),

$$u = \frac{f}{k} \left\{ Qx - Qa - \frac{P^2 \eta}{4\pi E} \left[x \ln \left(\frac{x}{a} \right) - x + x_1 \right] \right\} \quad (B10)$$

We may now obtain an average temperature for imaginary condition (1) as

$$\bar{T}_a = \frac{\int_{x_1}^{x_2} u \, dQ}{\int_{x_1}^{x_2} dQ} \quad (B11)$$

with use of equation (B10) for u and equation (B7) for dQ . Thus,

$$T_a = \frac{f}{k} \left[-2x_1 + \frac{2(x_2 - x_1)}{\ln\left(\frac{x_2}{x_1}\right)} - x_1 \ln\left(\frac{x_2}{x_1}\right) \right] \quad (B12)$$

But the temperature reached at depth x_e under imaginary condition (2) must be

$$T_b = \frac{f}{k} Q x_e \quad (B13)$$

Substituting from equation (B8) into equation (B13), then equating \bar{T}_a and T_b , and solving equations (B12) and (B13), we get

$$x_e = -\frac{2x_1}{\ln\left(\frac{x_2}{x_1}\right)} + \frac{2(x_2 - x_1)}{\ln^2\left(\frac{x_2}{x_1}\right)} - x_1 \quad (B14)$$

As we must not make x_1 equal to zero, probably a minimum reasonable value for it would be about half the diameter of a carbon atom or about 9×10^{-8} mm.

For use in estimating a reasonable value for x_2 , we will assume that, whenever a rupture of either a rubber-to-aluminum bond or a rubber-to-rubber bond occurs, the rubber atom will not have been displaced by more than about 10 diameters of a carbon atom, or about 1.8×10^{-6} mm. We will assume that the rubber is sliding relative to the aluminum at 30 m/sec (98.4 ft/sec).

We may compute the speed of a transverse (or a solenoidal) wave through the tread rubber with the following equation from reference 12:

$$v = \sqrt{\frac{G}{\rho}} \quad (B15)$$

where G is shearing modulus of elasticity, or rigidity. With a value range of 1.3×10^6 to 2.0×10^6 N/m² for G (values for polyisoprene vulcanized with 33 percent carbon black

from ref. 13) and with $\rho = 1200 \text{ kg/m}^3$ (ref. 14), equation (B15) yields a value range for v of 32.9 to 40.8 m/sec. We will use the approximate median of this range, 37 m/sec.

The time for travel through 10 diameters of a carbon atom at 30 m/sec (98.4 ft/sec) would be 6×10^{-11} sec. During that time, the transverse wave at 37 m/sec would travel approximately 2.2×10^{-6} mm. Thus, 2.2×10^{-6} mm seems to be a reasonable value for x_2 .

With the values $x_1 = 9 \times 10^{-8}$ mm and $x_2 = 2.2 \times 10^{-6}$ mm, equation (B14) gives us an effective depth of heat generation of $x_e = 2.66 \times 10^{-7}$ mm. This result puts x_e within less than two diameters of a carbon atom from the rubbing surface. The resistance to the flow of the heat through two layers of atoms or less should be quite negligible.

Clark and Staples conclude from equation (B8) and experiment that much more heat would be generated in rubber than in aluminum when one slides over the other. This author would not for a moment quarrel with that conclusion. In the discussion following equation (1) he stated that it represents an imaginary equivalent condition in which heat generation occurs on the surfaces in proportion to their conductivities. No such real condition was assumed or concluded to exist in the derivation of the equation in reference 1. Instead, under the assumption that the heat was generated practically within the surface layers of atoms, it was considered that it could flow from the surface of one asperity to the surface of another asperity (in the other material) with only negligible contact resistance. The result is the same as if the heat had been generated at the roots of the asperities in proportion to the conductivities.

Gross stick-slip conditions unquestionably exist. Such conditions are necessary, for example, to explain the jagged curves for rate of heat generation in figures 9 to 12. Stick-slip is thought to be due to the fact that static friction is usually greater than sliding friction. The stick-slip condition is usually associated with a vibratory system that involves oscillation of all or part of the rubbing surface fore and aft in the direction of sliding. The vibration may be far from sinusoidal. Whatever the velocity of the slide, a vibrational amplitude might exist at which the maximum backward velocity of the surface due to the vibration is equal to the forward velocity of the slide. Static friction then exists momentarily once during each cycle of oscillation and causes energy to be fed into the vibratory system. The stick-slip condition terminates when the sliding velocity requires an excessive oscillatory amplitude, which ruptures part of the vibrating system, makes it very nonlinear, or causes it to dissipate energy more rapidly than the static friction can feed energy into it.

The stick-slip condition shown by the curves for rate of heat generation in figures 9 to 12 must surely have caused some heat generation deep within the rubber. That heat, however, would not have had time to flow appreciably through the rubber to the temperature sensors. Consequently, it could not have formed an appreciable part of

the 64 percent of total heat estimated to be caused by the type of slippage known as squirming. It seems, therefore, that little leeway exists for the supposition of a substantial amount of heat generation by the gross stick-slip condition. Only 36 percent of the total drag of the tire was ascribed to the effect of hysteresis. The minute distortions of the rubber caused by sticking of elements of its surface several times during a total movement of only 1.5 mm could scarcely compare with the gross cyclic distortions of the rubber caused by the large flat spot in contact with the supporting surface. Hence the heat generation within the rubber due to the stick-slip action could be only a small part of the 36 percent.

From the foregoing analysis and discussion, it appears that the treatment of this subject by Clark and Staples, in conjunction with the analysis presented here, has helped considerably to demonstrate that heat of friction is generated mainly within the surface atoms of rubber and that it should be possible to dissipate it rapidly from that surface according to equation (1) with use of a supporting material of high thermal conductivity. (I am informed that the first author of ref. 3 does not agree with this statement. I do not know whether the second author is yet aware of the statement.) Additional evidence from their report as to the efficacy of an aluminum surface for that purpose will now be discussed.

DISSIPATION OF FRICTIONAL HEAT FROM RUBBER TO HIGHLY CONDUCTIVE MATING SURFACE

Clark and Staples in reference 3 report a summary of tests involving the sliding of a tire with locked wheel on a strip of aluminum sheet approximately 6.1 m (20 ft) long and 0.48 mm (0.019 in.) thick. They report the area of the contact patch as 380.6 cm² (59 in.²) and the length as 30 cm (11.8 in.). The width of the contact patch, then, should have been 12.7 cm (5 in.). They report that a temperature rise of 44.4 K (80° F) was measured on the surface of the contact patch at the end of the slide, with a load of 4450 N (1000 lb) and velocity of 0.643 m/sec (2.11 ft/sec).

It is known (ref. 7) that substantially uniform temperature will be reached throughout the depth of a layer of metal, with a constant heat flux through the surface, if

$$\frac{\alpha t}{l^2} > 1 \quad (B16)$$

where l is thickness. For aluminum, with diffusivity of 8.58×10^{-5} m²/sec, contact duration of 0.466 sec, and $l = 0.48$ mm, this criterion was far exceeded in the tests reported by Clark and Staples with $\alpha t / l^2 = 174$.

With the normal load of 4450 N and coefficient of friction of 1.4, the work done against friction W_f for each meter of slide was 6230 J. The mass of aluminum heated during 1 meter of slide must have been the product of that meter by the width w of the contact patch, the thickness l of the aluminum sheet, and the density of the aluminum ρ_{al} . The temperature elevation then, under the optimistic assumption that all the heat generated by friction went into the aluminum, would have been

$$\Delta T = \frac{W_f}{w l c_{al} \rho_{al}} \quad (B17)$$

With the specific heat $c_{al} = 865 \text{ J/(kg)(K)}$ and $\rho_{al} = 2700 \text{ kg/m}^3$, equation (B17) gives a temperature rise of 43.8 K.

This result agrees remarkably well with the temperature rise of about 44.4 K on the tread surface reported by Clark and Staples. Under the assumption that virtually all the heat flows into the aluminum, with very high contact conductance, the maximum temperature of the rubber surface (at the trailing end of the contact patch) immediately after the slide should be only slightly higher than the temperature reached by the aluminum. The fact that the maximum temperature reached by the rubber surface was just 0.6 K higher than the calculated temperature that should have been reached by the aluminum is another verification of the theory that the heat of friction is generated almost wholly on the surface of the rubber and can flow almost entirely into a highly conductive supporting surface. This result also indicates that thermal contact resistance between tread rubber and aluminum is not great enough to be a substantial impediment to the dissipation of most of the friction heat into the aluminum. The result also supports the earlier assumption that the endothermic rupture of rubber-to-rubber bonds does not account for a substantial fraction of the energy consumed.

Note that the temperature rise of the rubber surface is theoretically not affected by the velocity of the slide so long as the criterion (B16) is satisfied. With thicker aluminum surfaces, however, substantial heat could be conducted away from the rubber even if this criterion were far from satisfied.

Also note, in view of the close agreement of the tire surface temperature rise reported by Clark and Staples with the calculated temperature rise for the aluminum from equation (B17), that the tire could have been slid much farther without substantially greater rise of its surface temperature.

THERMAL CONTACT CONDUCTANCE

Clark and Staples in reference 3 estimated a value of thermal contact resistance. They designated their result as R_* and called it a "heat flux impedance coefficient",

emphasizing that "it is a measure of contact resistance but it is not the contact resistance."

They heated a part of the tread surface of a stationary tire greater than the anticipated contact area with heat lamps. They then repeatedly rolled the heated portion of the tread over temperature sensors, making records of the resulting temperature histories of the sensors. From the rates of temperature rise of the sensors during the early part of the contact, with use of independent calibrations of the sensors, they deduced values of R_* .

Their results for R_* for two independent sensors were equivalent to conductances of 1.4×10^4 and 2.4×10^4 W/(m²)(K) or 2.4×10^3 and 4.2×10^3 Btu/(ft²)(hr)(°F). It was tempting for this author simply to point out that these values are within the range of possible contact conductances that he found from his study of figure 13. But after careful study of the sensor design, the method of test, and the results reported in reference 3, he regretfully concluded that this method cannot indicate the contact conductance with sufficient accuracy to be of much value.

The sensors used by Clark and Staples were aluminum disks to the under sides of which had been cemented resistance thermometers just like that illustrated in figure 1. Each disk was isolated from contact with anything but the tread surface, with the exception of a hollow plastic cylindrical pedestal, which supported the periphery of the aluminum disk from underneath.

This author subjected the system (tread rubber-aluminum disk-resistance thermometer) to runs of the same type of finite-difference computer program that has been described. He used a series of assumed values of contact conductance between tread rubber and aluminum disk. He assumed the aluminum disk and the resistance thermometer to be at a uniform temperature before contact, and assumed the tread rubber to be at a uniform temperature 40 K (72° F) higher. The computer program yielded rates of temperature rise during the first 50 to 100 msec for the nickel grid within the resistance thermometer.

For an assumed infinite contact conductance, the result was 71.1 K/sec (128° F/sec). For the same initial temperature difference of 40 K (72° F), Clark and Staples "sensor 2" showed a rate of temperature rise of 67.3 K/sec (121° F/sec). Thus, according to the finite-difference program, their sensor 2 showed an almost infinite contact conductance. For an assumed contact conductance of 3500 W/(m²)(K), the computer program showed a temperature rise of 40 K/sec (72° F/sec). Clark and Staples "sensor 1" showed a rate of 39.9 K/sec (71.8° F/sec). Hence, according to the finite-difference program, their sensor 1 indicated a contact conductance of only 3500 W/(m²)(K).

The rates of temperature rise from the computer program should both be expected to be too high, because of it was necessary to ignore heat that would flow from the aluminum disk to the supporting plastic pedestal. That is, if the computer program could

have yielded the correct rates of temperature rise for the assumed values of contact conductance, then somewhat higher contact conductances would have had to be assumed to match the rates found by Clark and Staples. It is believed this effect would be very small.

In the treatment of the results by Clark and Staples, it apparently was necessary for them to regard the thermal contact resistance as being the only impediment to heat flow for a period of 50 to 100 msec. Actually, in far less than 50 msec, substantial temperature gradients develop within the rubber, the aluminum disk, and the resistance thermometer. Because of these gradients, the actual temperature difference available to drive heat through the contact resistance between rubber and aluminum drops to a low value in much less than 50 msec and the heat flux becomes largely a function of the gradients rather than a function of contact resistance. This fact is clearly demonstrated by the computer result indicating a temperature rise rate for the nickel grid of only 71.1 K/sec (128° F/sec) even with infinite contact conductance.

The enormous difference between the conductances found from the two sensors with use of the finite difference program precludes the placing of much value on the results. This author is at a loss to explain the difference, except possibly with the suggestion either that sensor 1 did not have a good thermally conductive bond between the aluminum disk and the resistance thermometer, or that sensor 1 had a much thicker layer of cement between the disk and the thermometer than did sensor 2. (The conductance of the bond between the aluminum disk and the resistance thermometer used in the computer program was $4800 \text{ W}/(\text{m}^2)(\text{K})$, the same as earlier found for each of the bonds between polyimide and nickel grid.) Note that no such possible failure to achieve a good thermally conductive bond between sensors and asphalt for the tests reported in reference 2 could have affected the analysis of those test results that has been reported here. Even with the assumption of infinite contact conductance between the resistance thermometers and the asphalt, the finite-difference program showed only negligible heat entering the asphalt from the polyimide of the resistance thermometer.

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TABLE I. - TOTAL RATE OF HEAT FLOW THROUGH
BOTH INNER AND OUTER SURFACES OF TIRE
TREAD AT VARIOUS ELAPSED TIMES
AFTER START OF TEST

j	Elapsed time, sec	m_{1j} , K/m	m_{2j} , K/m	Rate of flow of heat out of tread, H_{oj} , W
0	0	---	---	0.0 (assumed)
1	40	445	130	45.16
	70			28.0 (by double interpolation)
2	100	667	348	80.4
3	180	778	1000	143.0
4	240	778	1610	193.6
5	480	1220	1350	206.2
6	720	1000	1850	230.6
7	960	1160	2000	255.4
8	1200	778	2580	274.0
9	1440	1390	2580	321.2

TABLE II. - WHEEL DRAG FORCES NECESSARY TO PROVIDE ENERGY DISSIPATED
BY TREAD HYSTERESIS AT VARIOUS TIMES AFTER START OF TEST

AT 22.4 METERS PER SECOND

i	Elapsed time, sec	Rate of flow of heat out of tread, H_{oi} , W	Median of time inter- val i (from start of test), sec	Average tempera- ture rise, ΔT_i , K	Length of time in- terval i, sec	Rate of tread hysteretic heat genera- tion, H_{hi} , W	Hysteretic drag of tread	
							N	lb
1	40	22.6	20	4.0	40	1680	75.16	16.90
2	100	62.8	70	4.5	60	1306	58.43	13.13
3	180	112.0	140	5.4	80	1231	55.08	12.38
4	240	168	210	4.4	60	1384	61.92	13.92
5	480	200	360	7.8	240	739	33.06	7.44
6	720	218	600	4.8	240	550	24.61	5.53
7	960	243	840	2.5	240	416	18.61	4.18
8	1200	265	1080	1.4	240	362	16.20	3.64
9	1440	298	1320	1.6	240	409	18.30	4.11

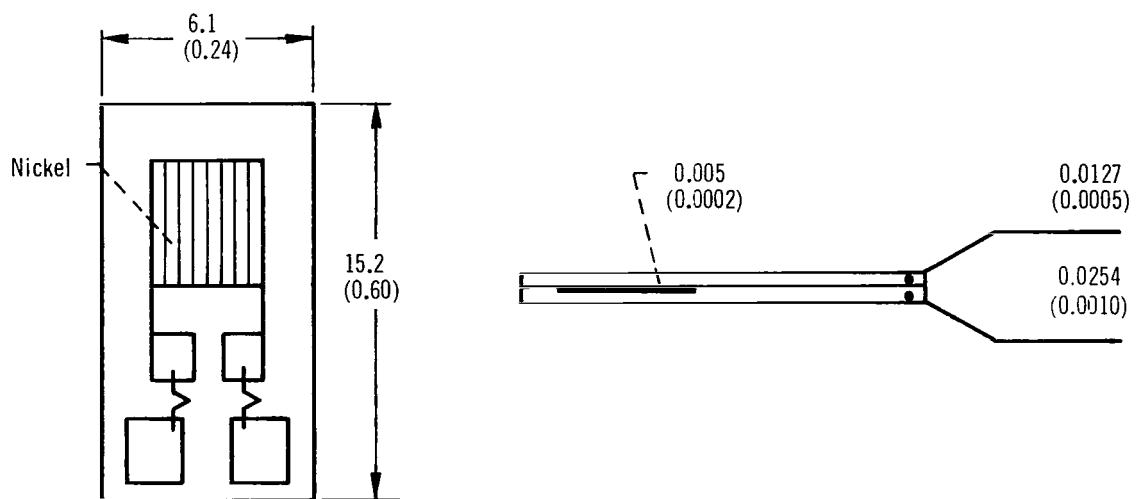


Figure 1. - Detail of sensor construction. (All dimensions in mm (in.).)

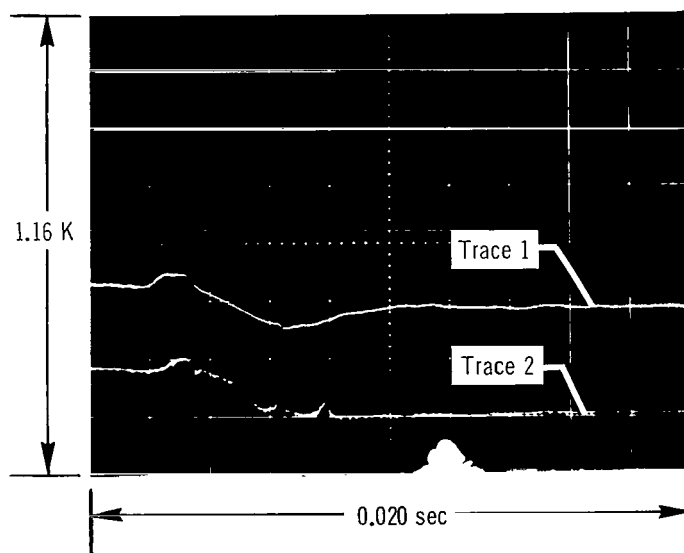


Figure 2. - Temperature-time records shown by two nickel-grid sensors when traversed by tire that had been in contact with pavement for 30 meters or less (ref. 2).

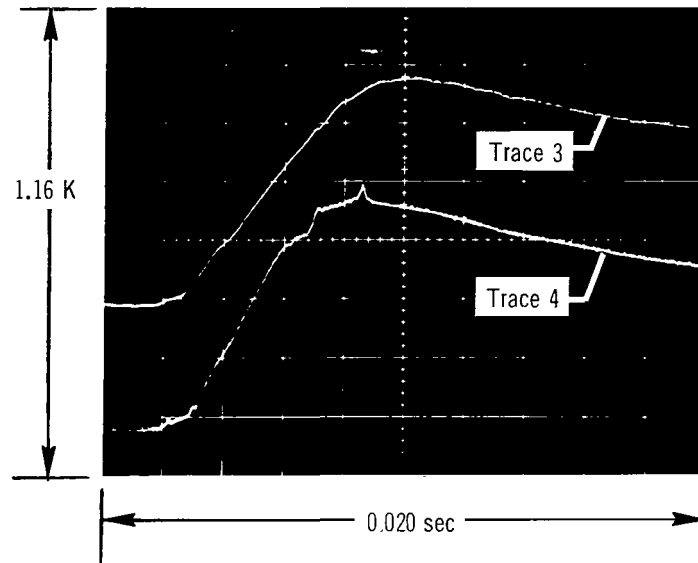


Figure 3. - Temperature-time records shown by two nickel-grid sensors when traversed by tire that had been rolling under load for 2 to 3 kilometers (ref. 2).

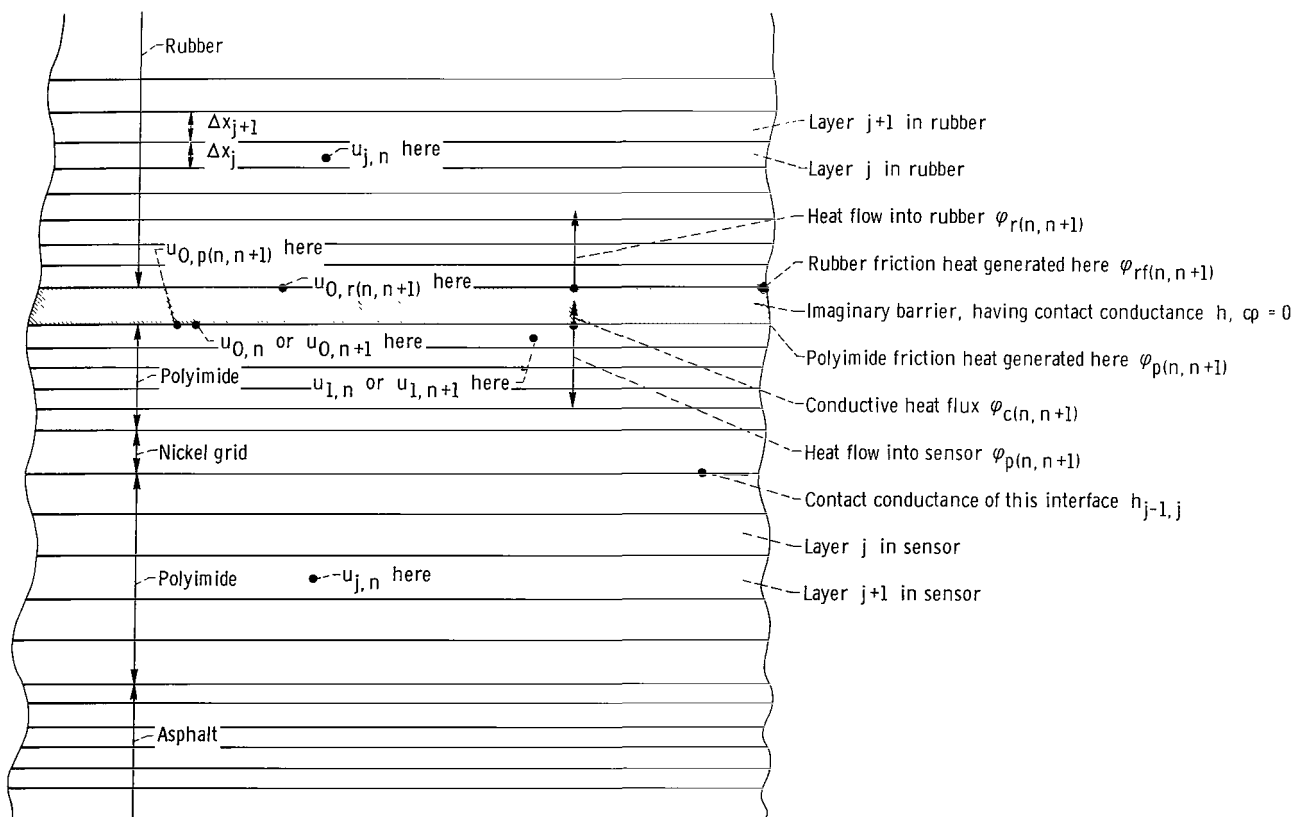


Figure 4. - Generation of heat within interface between surface of rubber tire and surface of temperature sensor, flow of heat, and entities used in numerical analysis.

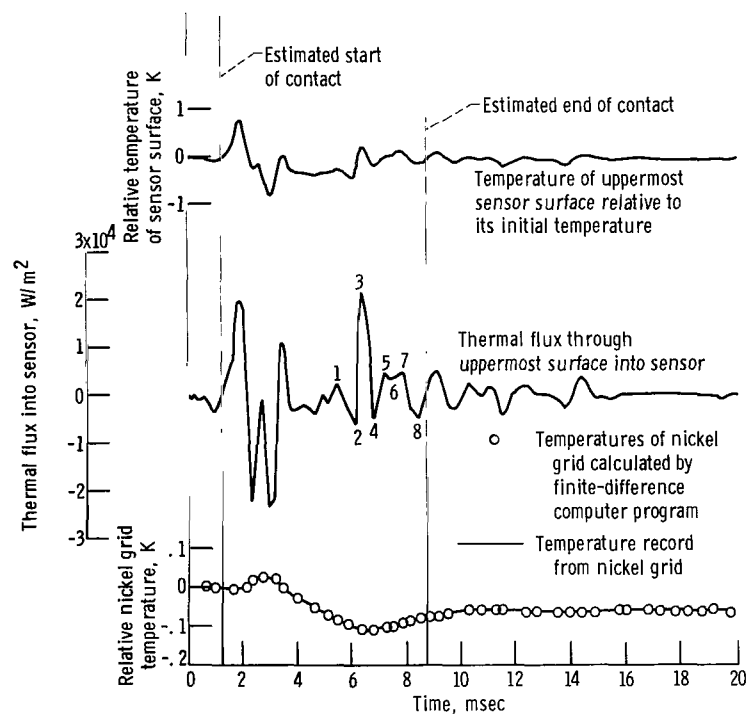


Figure 5. - Temperatures of upper surface and thermal fluxes into upper surface of nickel-grid sensor derived from temperature record of nickel grid for trace 1, in figure 2.

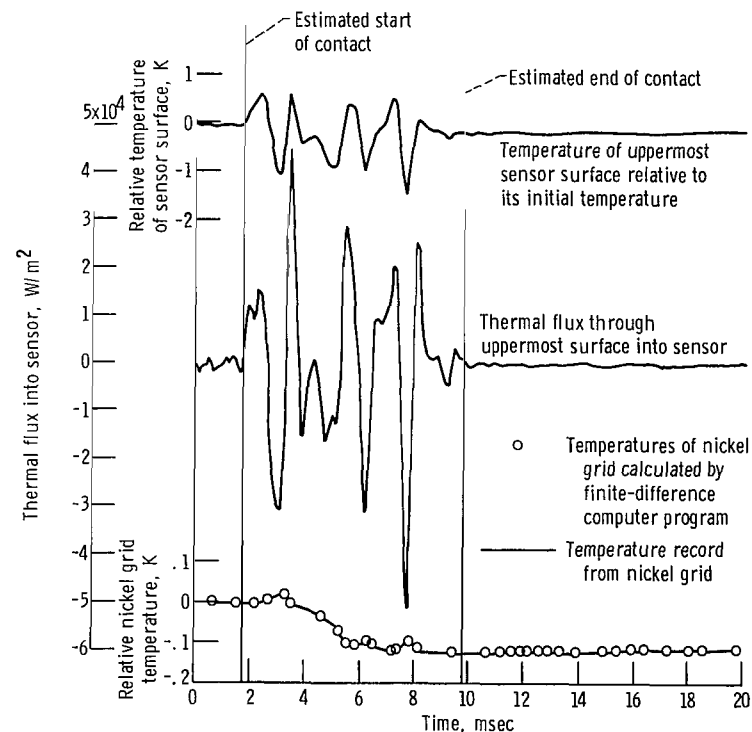


Figure 6. - Temperatures of upper surface and thermal fluxes into upper surface of nickel-grid sensor derived from temperature record of nickel grid for trace 2, in figure 2.

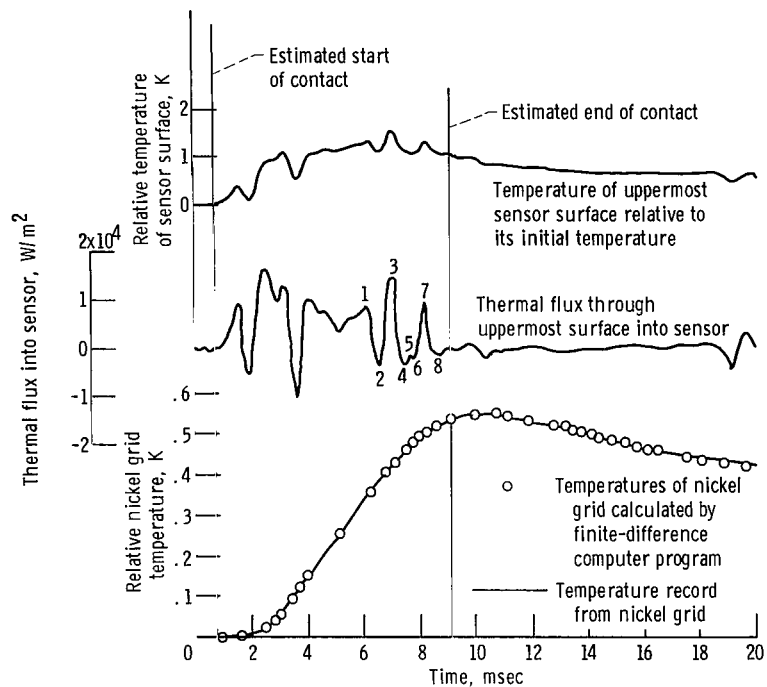


Figure 7. - Temperatures of upper surface and thermal fluxes into upper surface of nickel-grid sensor derived from temperature record of nickel grid for trace 3, in figure 3.

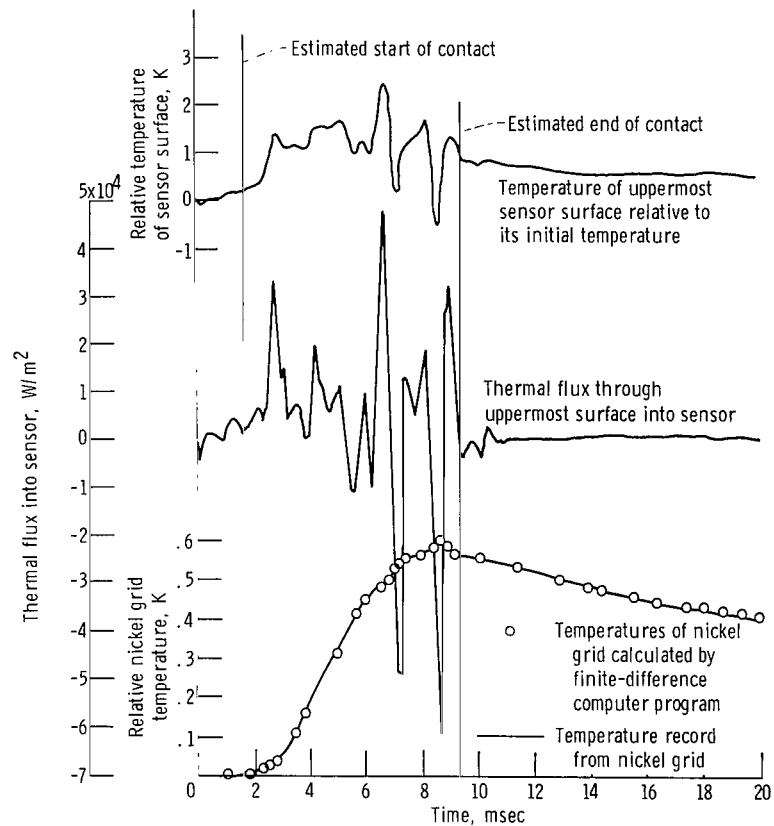


Figure 8. - Temperatures of upper surface and thermal fluxes into upper surface of nickel-grid sensor derived from temperature record of nickel grid for trace 4, in figure 3.

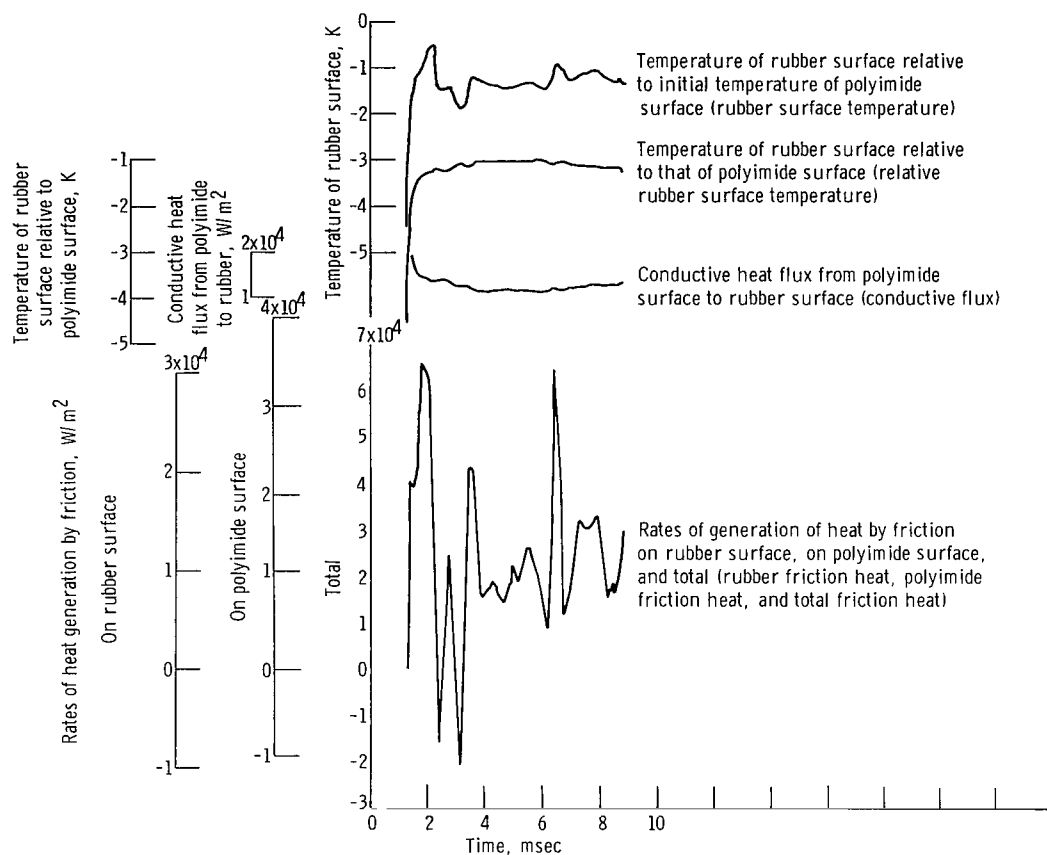


Figure 9. - Temperatures of rubber surface, conductive heat fluxes across polyimide-rubber interface, and rates of generation of heat by friction for trace 1, with assumed contact conductance of $11\,345\,W/(m^2)(K)$ or $2000\,Btu/(ft^2)(hr)(^{\circ}F)$.

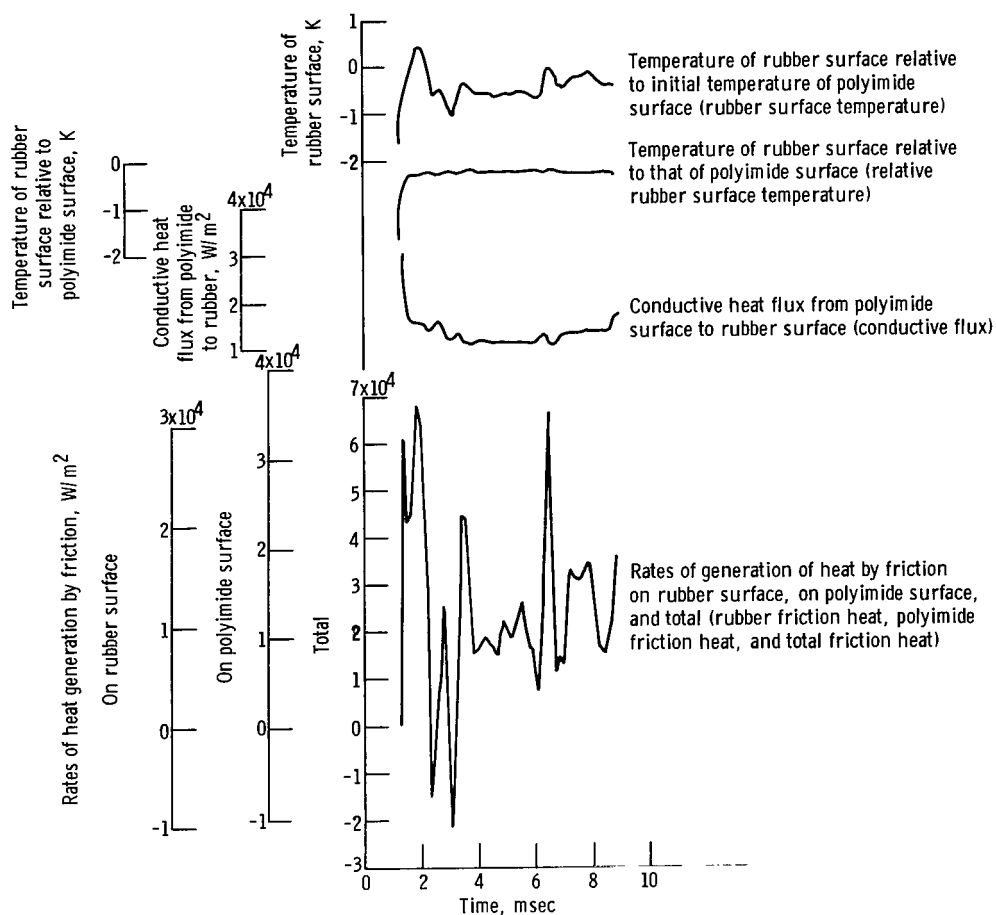


Figure 10. - Temperatures of rubber surface, conductive heat fluxes across polyimide-rubber interface, and rates of generation of heat by friction for trace 1, with assumed contact conductance of $56\,744\,W/(m^2)(K)$ or $10\,000\,Btu/(ft^2)(hr)(^{\circ}F)$.

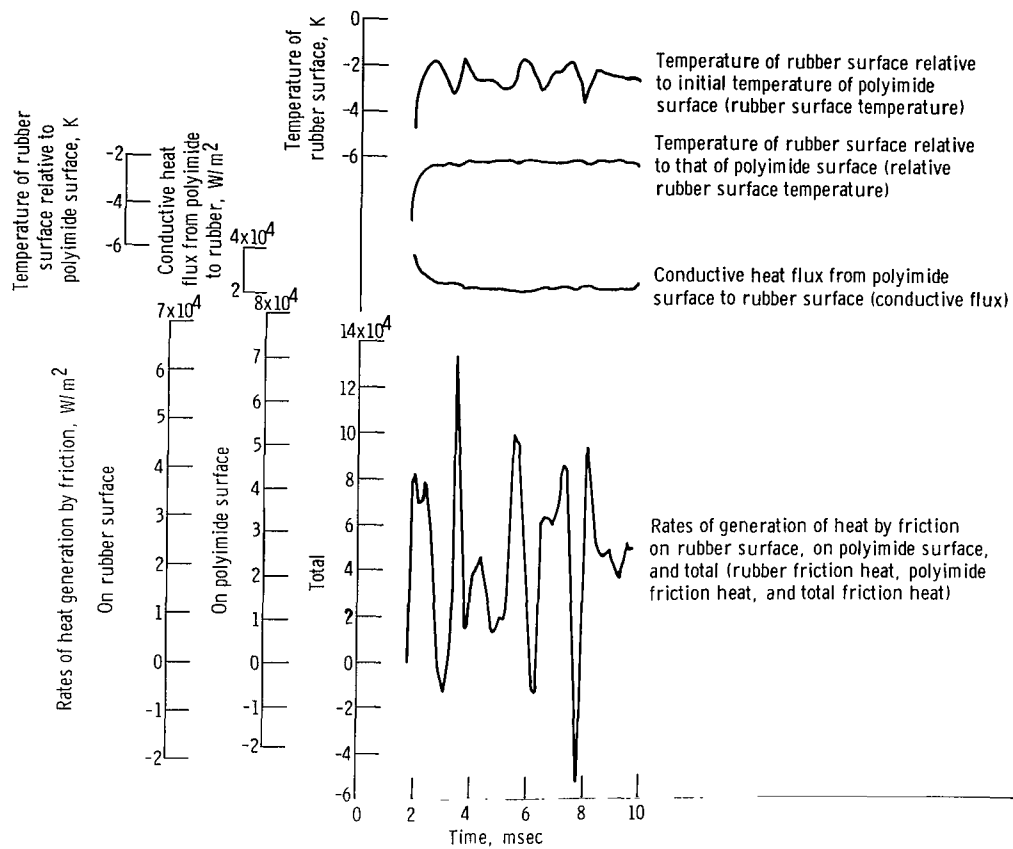


Figure 11. - Temperatures of rubber surface, conductive heat fluxes across polyimide-rubber interface, and rates of generation of heat by friction for trace 2, with assumed contact conductance of $11\,345\text{ W/(m}^2\text{)(K)}$ or $2000\text{ Btu/(ft}^2\text{)(hr)(}^\circ\text{F)}$.

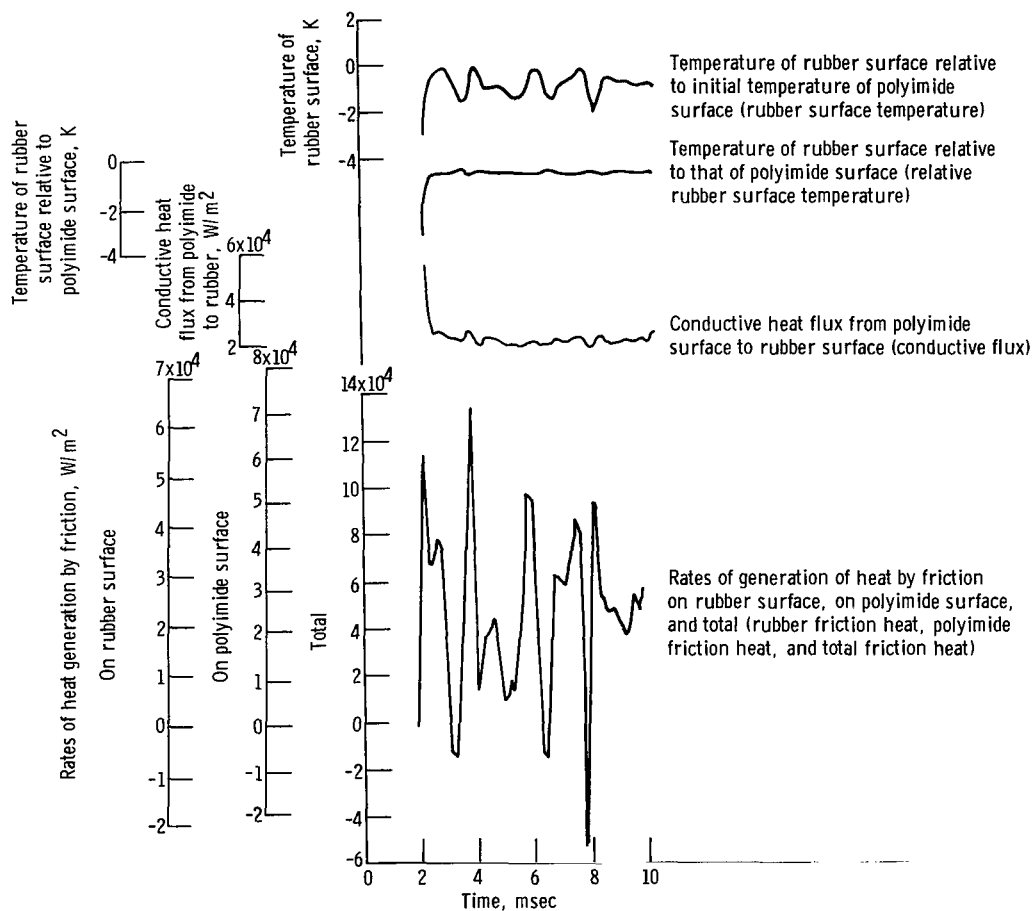


Figure 12. - Temperatures of rubber surface, conductive heat fluxes across polyimide-rubber interface, and rates of generation of heat by friction for trace 2, with assumed contact conductance of $56\,744\, W/(m^2)(K)$ or $10\,000\, Btu/(ft^2)(hr)(^{\circ}F)$.

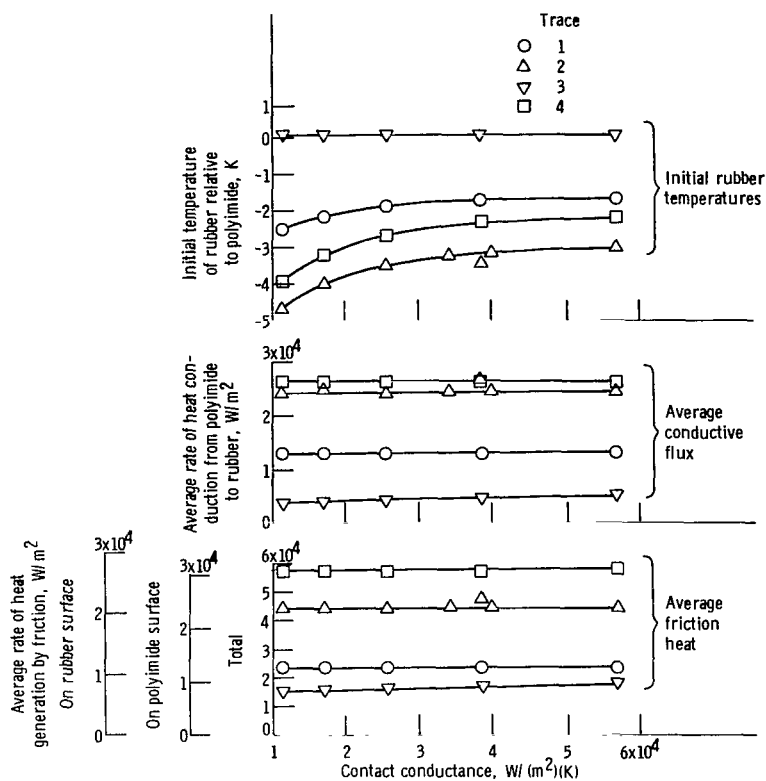


Figure 13. - Calculated initial rubber temperatures, rates of heat conduction, and rates of heat generation by friction for assumed values of contact conductance through interface between polyimide and rubber.

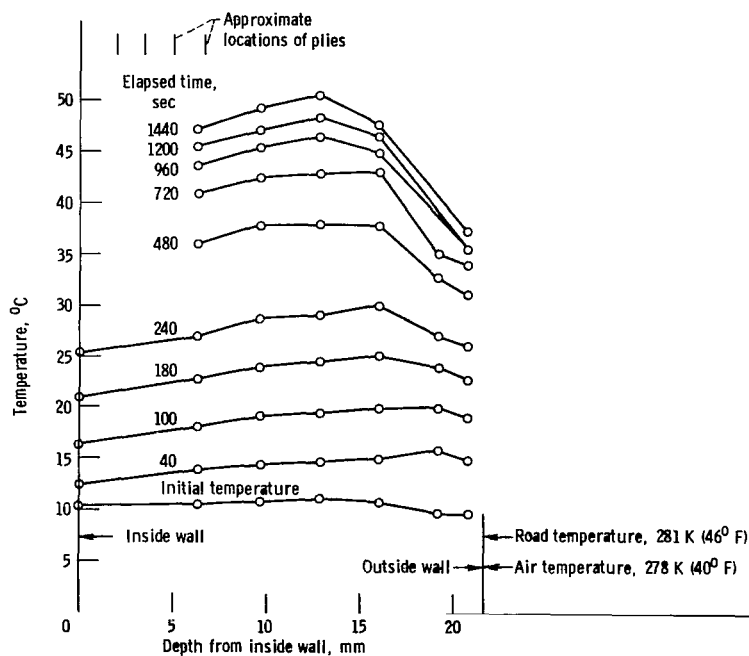


Figure 14. - Temperature profiles at crown.

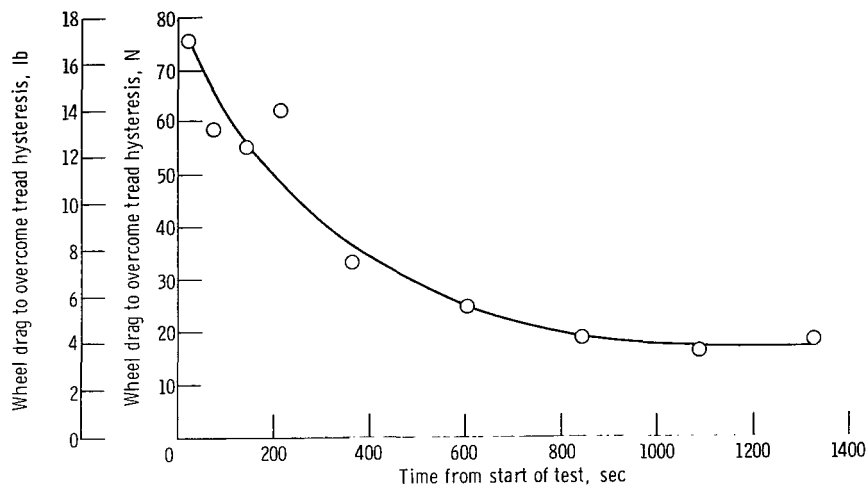


Figure 15. - Calculated values of wheel drag required to provide energy dissipated by mechanical hysteresis in tread rubber.

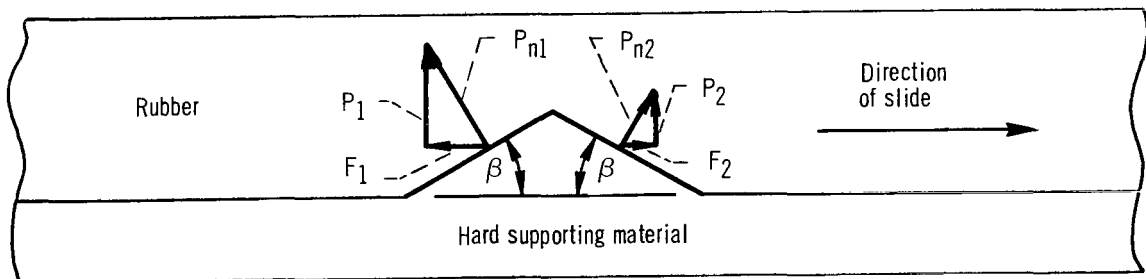


Figure 16. - Forces caused by asperity on rubbing surface.

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